Pr. 1 $f(x)=\dfrac{x^2}{x+1}$ $\qquad D_f = R - \{-1\}$

$$f'(x) = \frac{2x(x+1)-x^2}{(x+1)^2} = \frac{2x^2+2x-x^2}{(x+1)^2} = \boxed{\frac{x^2+2x}{(x+1)^2}}$$

$f'(x) = 0 \iff x(x+2)=0$

$x_1 = 0 \ \vee \ x_2 = -2$

f': $+$ $\ -2\ $ $-$ $\ -1\ $ $-$ $\ 0\ $ $+$

f: $\nearrow$ $\ -2\ $ $\searrow$ $\ -1\ $ $\searrow$ $\ 0\ $ $\nearrow$

$f'(1) = \frac{3}{4} > 0$

$f'(-\frac{1}{2}) = \dfrac{\frac{1}{4}-1}{\frac{1}{4}} < 0$

$f'(-\frac{3}{2}) < 0$

$f'(-10) = \frac{80}{81}$

Rastúca na $(-\infty,-2\rangle$, $\langle 0,\infty)$

Klesajúca na $\langle -2,-1)$, $(-1,0\rangle$.

$x_2 = -2$ bod OLMAX $\qquad f(-2) = -4$

$x_1 = 0$ bod OLMIN $\qquad f(0) = 0$

$$f''(x) = \frac{(2x+2)(x+1)^{\cancel{2}\,1} - (x^2+2x)\cdot 2\cancel{(x+1)}\cdot 1}{(x+1)^{\cancel{4}\,3}} = \frac{\cancel{2x^2}+\cancel{2x}+\cancel{2x}+2-\cancel{2x^2}-\cancel{4x}}{(x+1)^3} = \frac{2}{(x+1)^3}$$

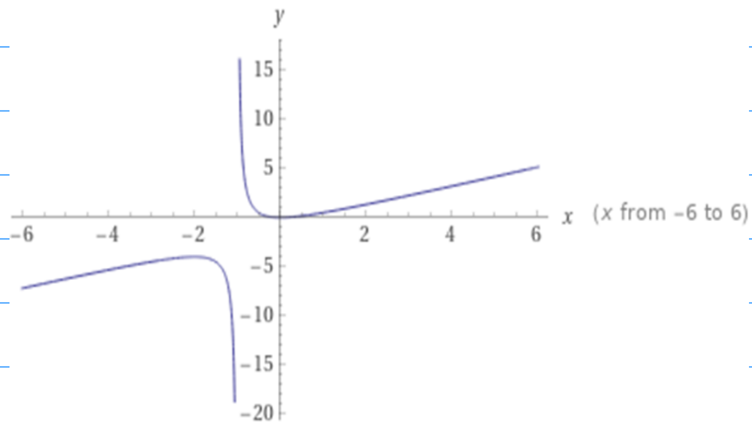
$f''(x) \neq 0$

f'': $-$ $\ -1\ $ $+$

$\frown$ $\qquad \smile$

$f''(0) = 2 > 0$

$f''(-2) = -2 < 0$



Konkávna na $(-\infty,-1)$

konvexná na $(-1,+\infty)$

Pr 2. $f(x) = \frac{x-1}{\sqrt{x^2+1}}$ $D_f = R$

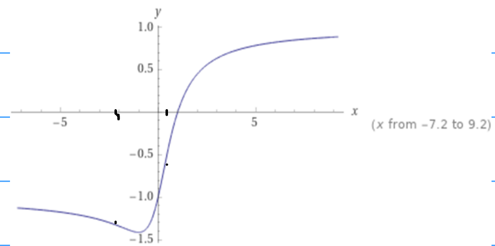
$$f'(x) = \frac{\sqrt{x^2+1} - (x-1)\,\cancel{\tfrac{1}{2}}(x^2+1)^{-\frac{1}{2}}\cdot \cancel{2}x}{x^2+1} = \frac{\sqrt{x^2+1} - \frac{x-1}{\sqrt{x^2+1}}\cdot x}{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = \frac{x^2+1-(x-1)\cdot x}{(x^2+1)^{\frac{3}{2}}} = \frac{\cancel{x^2}-\cancel{x^2}+x+1}{(x^2+1)^{\frac{3}{2}}}$$

$f'(x)=0 \iff x+1=0$ $x_1=-1$ bod OLMIN $f(-1)=\frac{-2}{\sqrt{2}}=-\sqrt{2}$

f': $-$ on $(-\infty,-1)$ ↘, $+$ on $(-1,\infty)$ ↗

Rastúca na $\langle -1, \infty)$

klesajúca na $(-\infty, -1\rangle$



$$f''(x) = \frac{(x^2+1)^{\frac{3}{2}} - (x+1)\cdot\frac{3}{2}\cancel{(x^2+1)^{\frac{1}{2}}}\cdot 2x}{(x^2+1)^{\frac{5}{2}}} = \frac{x^2+1-(3x^2+3x)}{(x^2+1)^{\frac{5}{2}}} = \frac{1-3x-2x^2}{(x^2+1)^{\frac{5}{2}}}$$

$f''(x)=0 \iff -2x^2-3x+1=0$ $x_{1,2}=\frac{3\pm\sqrt{9+8}}{-4} = \frac{3\pm\sqrt{17}}{4}$ $x_1 \doteq -\frac{7}{4}$, $x_2 \doteq \frac{1}{4}$

f'': $-$ ∩ x_1 $+$ ∪ x_2 $-$ ∩

konkávna na $\left(-\infty, -\frac{3+\sqrt{17}}{4}\right\rangle, \left\langle -\frac{3-\sqrt{17}}{4}, \infty\right)$

konvexná na $\left\langle -\frac{3+\sqrt{17}}{4}, -\frac{3-\sqrt{17}}{4}\right\rangle$

Pr3. $f(x) = e^{-x^2}(x^2-3)$ $D_f = \mathbb{R}$

$$f'(x) = e^{-x^2}\cdot(-2x)\cdot(x^2-3) + e^{-x^2}\cdot 2x = e^{-x^2}(-2x^3+6x+2x) = e^{-x^2}(8x-2x^3)$$

$$f'(x) = 0 \iff 8x - 2x^3 = 2x(4-x^2) = 0$$

$x_1 = 0$ OLMIN, $x_2 = 2$ OLMAX, $x_3 = -2$ OLMAX

f': $+$ | -2 | $-$ | 0 | $+$ | 2 | $-$

f: $\nearrow$ $\searrow$ $\nearrow$ $\searrow$

Rastúca na $(-\infty,-2\rangle$; $\langle 0,2\rangle$

Klesajúca na $\langle -2,0\rangle$, $\langle 2,\infty)$

$f(0) = -3$ $f(2) = e^{-4} = f(-2)$

$$f''(x) = e^{-x^2}(-2x)(8x-2x^3) + e^{-x^2}\cdot(8-6x^2) = e^{-x^2}(8-6x^2-16x^2+4x^4) = 2e^{-x^2}(4-11x^2+2x^4)$$

$$f''(x) = 0 \iff 4 - 11x^2 + 2x^4 = 0 \qquad x^2 = z \qquad x = \sqrt{z}$$

$$4 - 11z + 2z^2 = 0 \qquad z_{1,2} = \frac{11 \pm \sqrt{121-32}}{4} = \frac{11 \pm \sqrt{89}}{4}$$

$z_1 = \frac{11+\sqrt{89}}{4}$ $x_1 = \frac{\sqrt{11+\sqrt{89}}}{2} \doteq 2{,}25$ $x_2 = -\frac{\sqrt{11+\sqrt{89}}}{2}$

$z_2 = \frac{11-\sqrt{89}}{4}$ $x_3 = \frac{\sqrt{11-\sqrt{89}}}{2} \doteq 0{,}6$ $x_4 = -\frac{\sqrt{11-\sqrt{89}}}{2}$

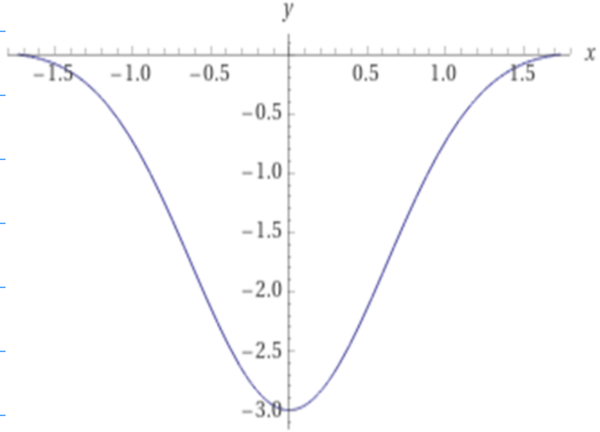
Inflexné body

$+$ | x_2 | $-$ | x_4 | $+$ | x_3 | $-$ | x_1 | $+$

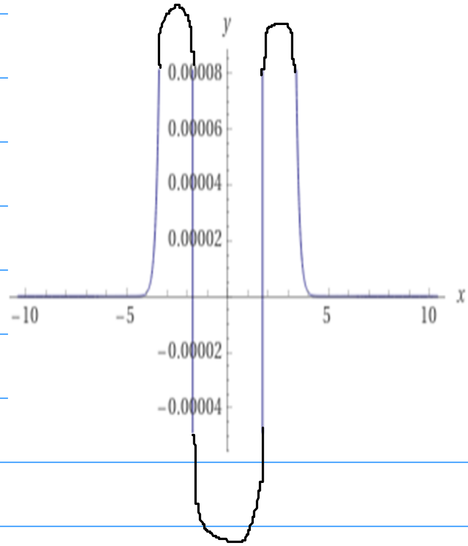
$\cup$ $\cap$ $\cup$ $\cap$ $\cup$

konvexná $(-\infty, x_2\rangle, \langle x_4, x_3\rangle, \langle x_1, \infty)$

konkávna $\langle x_2, x_4\rangle, \langle x_3, x_1\rangle$



(x from −1.7 to 1.7)



(x from −10.4 to 10.4)

l'Hospitalovo pravidlo

Pr 1.

$$\lim_{x\to 0}\frac{e^{3x}-1}{x} \overset{L'H}{=} \lim_{x\to 0}\frac{e^{3x}\cdot 3}{1} = 3 \qquad \lim_{x\to 0}\frac{e^{kx}-1}{x} = \lim_{x\to 0}\frac{e^{kx}\cdot k}{1} = k$$

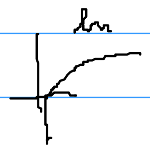
Pr 2.

$$\lim_{x\to 0+}\frac{\ln x}{\cot g\, x} \overset{L'H}{=} \lim_{x\to 0+}\frac{\frac{1}{x}}{-\frac{1}{\sin^2 x}} = \lim_{x\to 0+}\frac{-\sin^2 x}{x} =$$

$$„\frac{-\infty}{\infty}" \qquad „\frac{0}{0}"$$

$$(\cot g\, x)' = \frac{-1}{\sin^2 x}$$

$$(\cot g\, x)' = \left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin x\cdot\sin x - \cos x\cdot\cos x}{\sin^2 x}$$



$$= \lim_{x\to 0+} \frac{-2\sin x \cdot \cos x}{1} = \underline{\underline{0}}$$

Pr 3.

$$\lim_{x\to 0} \frac{x - \sin x}{x - x\cos^2 x} \overset{L'H}{=} \lim_{x\to 0} \frac{1 - \cos x}{1 - (\cos^2 x + x\,2\cos x\cdot(-\sin x))} = \lim_{x\to 0} \frac{1-\cos x}{1-\cos^2 x + x\sin 2x} =$$

$$\text{"}\frac{0}{0}\text{"} \qquad \text{"}\frac{0}{0}\text{"}$$

$2\sin\alpha\cos\alpha = \sin 2\alpha$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\sin x}{\underbrace{+2\cos x\cdot\sin x}_{\sin 2x} + (\sin 2x + x\cos 2x\cdot 2)} = \lim_{x\to 0} \frac{\sin x}{2\sin 2x + 2x\cos 2x} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{2\cos 2x\cdot 2 + 2\cos 2x + 2x(-\sin 2x)\cdot 2} =$$

$$\text{"}\frac{0}{0}\text{"}$$

$$= \frac{1}{4+2+0} = \frac{1}{6}$$