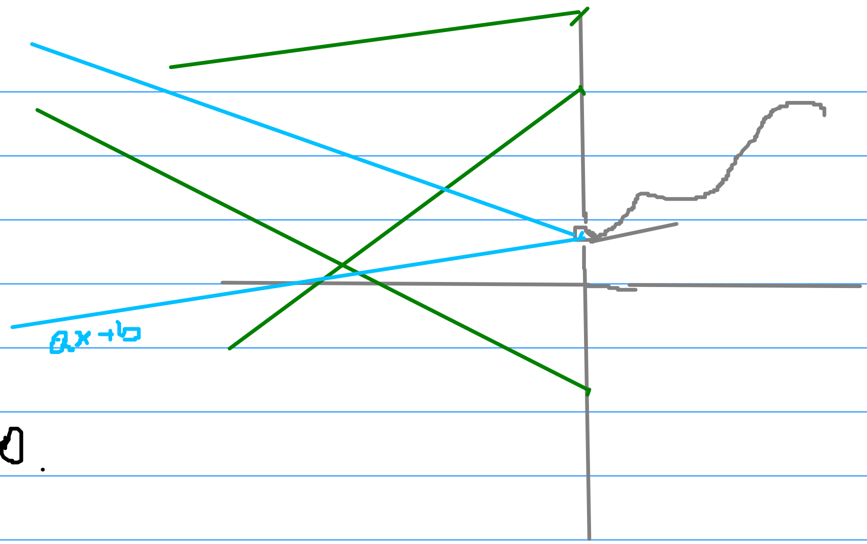
$$f(x) = \begin{cases} f_1(x) & x > 0 \\ ax+b & x \le 0 \end{cases}$$



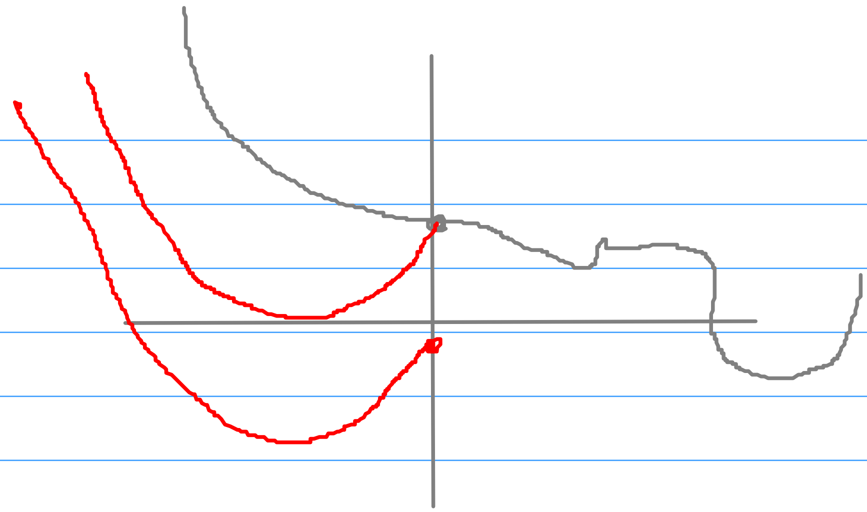
Nájdite hodnoty a, b tak, aby f bola <u>diferencovateľná</u> v bode 0.

$$\lim_{x\to 0+} f_1(x) = \lim_{x\to 0-} ax+b = b$$

$f'(x)$ pre $x > 0$ $\qquad f_1'(x)$

$$\lim_{x\to 0+} f_1'(x) = \lim_{x\to 0-} (ax+b)' = a$$

$$f(x) = \begin{cases} \frac{\sin x}{x} & x > 0 \\ x^2+ax+b & x \le 0 \end{cases}$$



Nájdite hodnoty a, b tak, aby

f bola diferencovateľná v bode 0

$$\lim_{x\to 0+} \frac{\sin x}{x} = 1 = \lim_{x\to 0-} x^2+ax+b = b \quad \Rightarrow b = 1 \text{ aby } f \text{ bola spojitá}$$

$$\lim_{x\to 0+} \frac{\cos x \cdot x - \sin x \cdot 1}{x^2} = \left(\lim_{x\to 0+} \frac{\frac{\cos x \cdot x - \sin x}{x}}{x} = \lim_{x\to 0+} \frac{\overset{\overset{1}{\uparrow}}{\cos x} - \overset{\overset{1}{\uparrow}}{\frac{\sin x}{x}}}{x} \right)$$

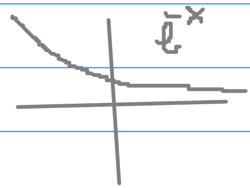
$$\overset{L'H}{=} \lim_{x\to 0+} \frac{-\sin x \cdot \cancel{x} + \cancel{\cos x} - \cancel{\cos x}}{2\cancel{x}} = 0 =$$

$$\lim_{x\to 0-} 2x + a = a \qquad a = 0$$

$$f(x) = \begin{cases} \frac{\sin x}{x} & x > 0 \\ x^2+1 & x \le 0 \end{cases}$$

Monotónnosť (a lokálne extrémy)

(1.) $f(x) = xe^{-x}$ $\qquad D_f = R$



$$f'(x) = 1 \cdot e^{-x} + xe^{-x} \cdot (-1) = e^{-x} \cdot (1-x)$$

Stac. bod : $x_0 = 1$ $\qquad e^{-x} > 0$

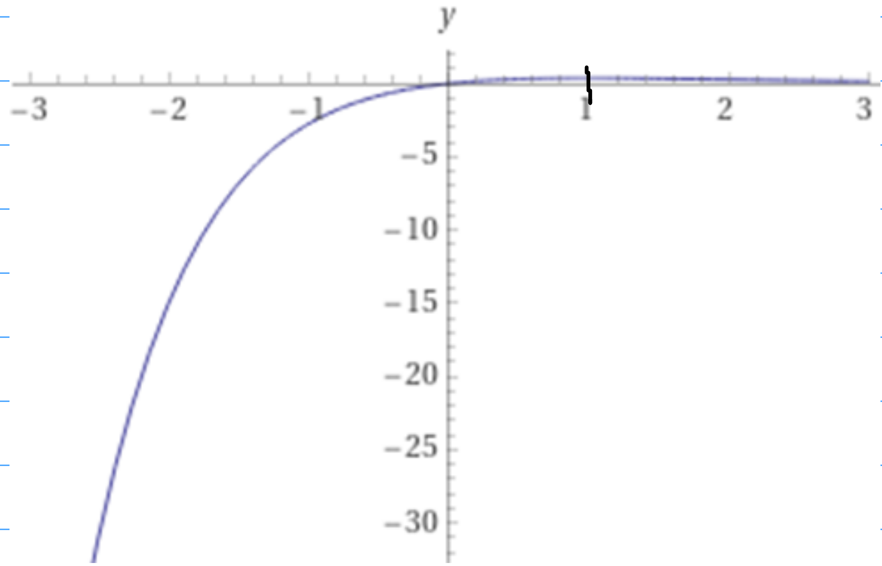
f': $+$ | $-$

f: $\nearrow$ 1 $\searrow$

f je rastúca na $(-\infty, 1\rangle$

f je klesajúca na $\langle 1, \infty)$

$x_0 = 1$ je bod OLMAX



(2.) $f(x) = \ln(x^2 - 2x + 2)$ $\qquad D_f = R$

$x^2 - 2x + 2 > 0$ $\qquad \Uparrow$

$(x^2 - 2x + 1) + 1 = (x-1)^2 + 1 > 0$

$$f'(x) = \frac{1}{x^2 - 2x + 2} \cdot (2x - 2) = \frac{2x-2}{x^2-2x+2}$$

$f'(x) = 0 \iff 2x - 2 = 0$

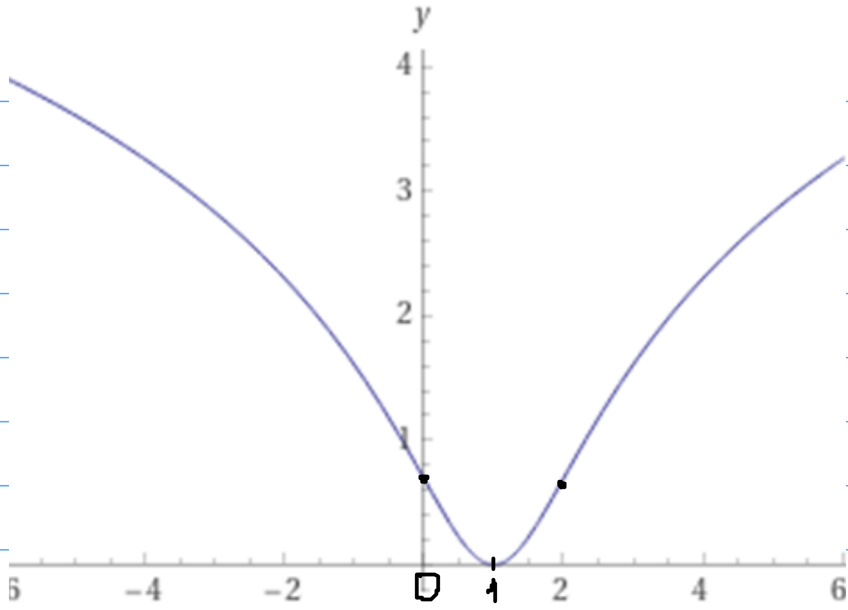
Stac. body: $x_0 = 1$

f': $-$ on $(-\infty, 1)$, $+$ on $(1, \infty)$

f: $\searrow$ $\quad 1 \quad$ $\nearrow$

Funkcia f je klesajúca na $(-\infty, 1\rangle$
rastúca na $\langle 1, \infty)$

Bod $x_0 = 1$ je bod OLMIN



$$f''(x) = \frac{2(x^2-2x+2) - (2x-2)\cdot(2x-2)}{(x^2-2x+2)^2} =$$

$$= \frac{2x^2-4x+4-[4x^2-8x+4]}{(x^2-2x+2)^2} = \frac{-2x^2+4x}{(x^2-2x+2)^2} = \boxed{-\frac{x(2x-4)}{(x^2-2x+2)^2}}$$

$f''(x) = 0 \qquad x_1 = 0 \qquad x_2 = 2$ inflexné body

$f''(1) = -\frac{1(-2)}{1^2} > 0$

$f''(10) = -\frac{10(16)}{82^2} < 0$

$f''(-1) < 0$

f'': $-$ $\quad 0 \quad$ $+$ $\quad 2 \quad$ $-$

f: $\cap$ $\quad 0 \quad$ $\cup$ $\quad 2 \quad$ $\cap$

konvexná na $\langle 0, 2\rangle$
konkávna na $(-\infty, 0\rangle$ a na $\langle 2, \infty)$.

③ $f(x) = \frac{x^3}{3-x^2}$ $\qquad D_f = R - \{-\sqrt{3}, \sqrt{3}\}$

$3 - x^2 = 0 \qquad x^2 = 3 \qquad x = \pm\sqrt{3}$

$f'(x) = \frac{3x^2(3-x^2) - x^3(-2x)}{(3-x^2)^2} = \frac{9x^2 - 3x^4 + 2x^4}{(3-x^2)^2} = \frac{(9-x^2)x^2}{(3-x^2)^2}$ $\qquad D_{f'} = R - \{-\sqrt{3}, \sqrt{3}\}$

Párna

St. body $(9-x^2)x^2 = 0$ $\qquad x_1 = 0, \; x_2 = 3, \; x_3 = -3$

$f'(1) = 2 > 0$

$f'(2) = \frac{20}{1} > 0$

$$\begin{array}{c|ccccccccccc}
f' & - & -3 & + & -\sqrt{3} & + & 0 & + & \sqrt{3} & + & 3 & - \\
f & \searrow & & \nearrow & & \nearrow & & \nearrow & & \nearrow & & \searrow
\end{array}$$

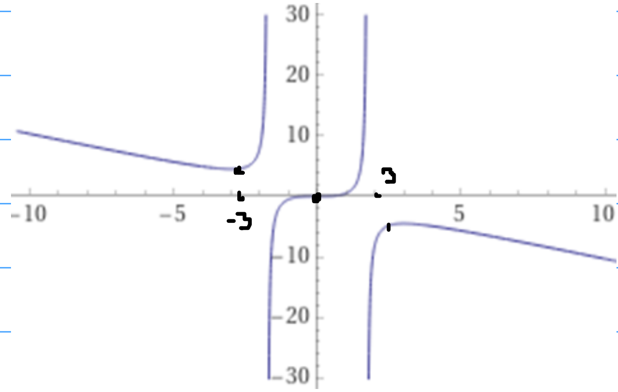
Klesajúca na $(-\infty, -3\rangle$ a $\langle 3, \infty)$

Rastúca na $\langle -3, -\sqrt{3})$, na $(-\sqrt{3}, \sqrt{3})$ a $(\sqrt{3}, 3\rangle$

$x_1 = 0$ nie je bod lok. extrému

$x_2 = 3$ bod OLMAX $\qquad f(3) = \frac{27}{-6} = -\frac{9}{2}$

$x_3 = -3$ bod OLMIN $\qquad f(-3) = \frac{9}{2}$



(4) $f(x) = \dfrac{x-1}{\sqrt{x^2+1}}$ $\qquad D_f = \mathbb{R}$

$$f'(x) = \frac{1\sqrt{x^2+1} - (x-1)\cdot\frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}\cdot\cancel{2}x}{x^2+1} = \frac{\sqrt{x^2+1} - \frac{x^2-x}{\sqrt{x^2+1}}}{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = \frac{x^2+1-(x^2-x)}{(x^2+1)^{\frac{3}{2}}} = \frac{1+x}{(x^2+1)^{\frac{3}{2}}}$$

Stac. bod $x_0 = -1$

f': $-$ on $(-\infty,-1)$ ↘, $+$ on $(-1,\infty)$ ↗

Rastúca na $\langle -1, \infty)$
Klesajúca na $(-\infty, -1\rangle$

x_0 je bod OLMIN $\qquad f(-1) = \dfrac{-2}{\sqrt{2}} = -\sqrt{2}$

konvexnosť

$$f''(x) = \frac{1\cdot(x^2+1)^{\cancel{\frac{3}{2}}\,1} - (1+x)\cancel{\frac{3}{2}}\cancel{(x^2+1)^{\frac{1}{2}}}\cdot\cancel{2}x}{(x^2+1)^{\cancel{3}\,\frac{5}{2}}} = \frac{x^2+1-3(x+x^2)}{(x^2+1)^{\frac{5}{2}}} = \frac{-2x^2-3x+1}{(x^2+1)^{\frac{5}{2}}}$$

$$f''(x)=0 \iff -2x^2-3x+1=0 \qquad x_{1,2} = \frac{3\pm\sqrt{9+4\cdot2\cdot1}}{-4} = -\frac{3}{4}\pm\frac{\sqrt{17}}{4} = \begin{cases} x_1 = \frac{-3+\sqrt{17}}{4} \\ x_2 = \frac{-3-\sqrt{17}}{4}\end{cases}$$

f'': $-$ | x_2 ($-\frac{7}{4}$) | $+$ | x_1 ($\frac{1}{4}$) | $-$

f: ∩ ∪ ∩

$f''(0) = \frac{1}{1} = 1 > 0$

$f''(1) = \dfrac{-4}{2^{\frac{5}{2}}} < 0$ $\qquad f''(-2) = \dfrac{-8+6+1}{5^{\frac{5}{2}}} < 0$

konvexná na $\left\langle \frac{-3-\sqrt{17}}{4}, \frac{-3+\sqrt{17}}{4} \right\rangle$

konkávna na $\left(-\infty, \frac{-3-\sqrt{17}}{4}\right\rangle$ a na $\left\langle \frac{-3+\sqrt{17}}{4}, \infty\right)$.

$x_1 = \frac{-3+\sqrt{17}}{4}, x_2 = \frac{-3-\sqrt{17}}{4}$ sú inflexné body.