$f(x) = \ln\sqrt{5-x}$ $\quad A = [?, 0]$

$\ln\sqrt{5-x} = 0$

$\sqrt{5-x} = e^0 = 1$

$5 - x = 1$

$x = 4 \quad A = [4, 0]$

$f'(x) = \frac{1}{\sqrt{5-x}} \cdot \frac{1}{2}(5-x)^{-\frac{1}{2}} \cdot (-1) =$

$f'(4) = \frac{1}{1} \cdot \frac{1}{2} \cdot (-1) = -\frac{1}{2}$

$y - 0 = -\frac{1}{2}(x - 4)$

$df(x, 4) = -\frac{1}{2}(x-4)$

Pr 1. $f(x) = x \cdot e^{-x}$ $D_f = R$

Stac. body $f'(x) = 1 \cdot e^{-x} + x e^{-x} \cdot (-1) = \underline{e^{-x}(1-x)}$ $f'(x) = 0 \iff x = 1$ $x_0 = 1$ St. bod

f'	$+$		$-$
		1	
f	$\nearrow$		$\searrow$

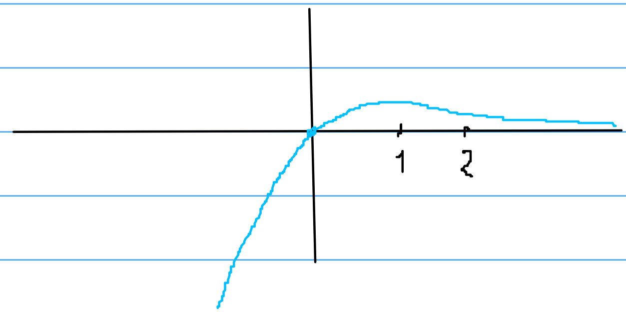
$f'(2) = e^{-2}(-1) = -\frac{1}{e^2} < 0 \Rightarrow f'(x) < 0$ na $(1, \infty)$.

$f'(0) = 1 > 0 \Rightarrow f'(x) > 0$ na $(-\infty, 1)$.

$x_0 = 1$ je bod OLMAX $f(1) = \frac{1}{e}$

f je rastúca na $(-\infty, 1\rangle$

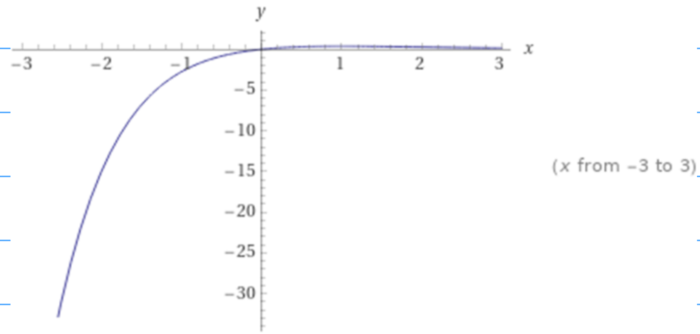
f je klesajúca na $\langle 1, \infty)$.



$$\lim_{x \to \infty} \frac{x}{e^x} = 0 \qquad \lim_{x \to -\infty} \frac{x}{e^x} = -\infty$$

$f''(x) = e^{-x} \cdot (-1)(1-x) + e^{-x}(-1) = e^{-x}(x-2)$ $f''(x) = 0 \iff x = 2$ $x_1 = 2$ inflexný bod

f''	$-$		$+$
		2	
f	$\frown$		$\smile$



Pr 2. $f(x) = \ln(x^2-2x+2)$ $\qquad x^2-2x+2 \quad D = 4-4\cdot 2 = -4 < 0$ nemá reálne korene

$x^2-2x+1+1 = (x-1)^2+1 > 0$

$D_f = \mathbb{R}$

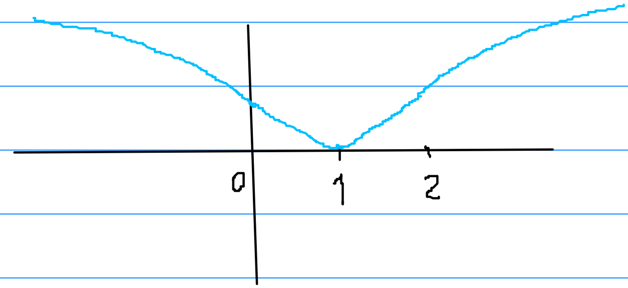
$f'(x) = \dfrac{1}{x^2-2x+2}\cdot(2x-2)$ $\qquad f'(x)=0 \iff x=1$ St. bod

$f'(2) > 0$

$f'(0) < 0$

f': $-$ | 1 | $+$

f: $\searrow$ | 1 | $\nearrow$



f je klesajúca na $(-\infty, 1\rangle$

rastúca $\langle 1, \infty)$. $\qquad x_0 = 1$ bod OLMIN

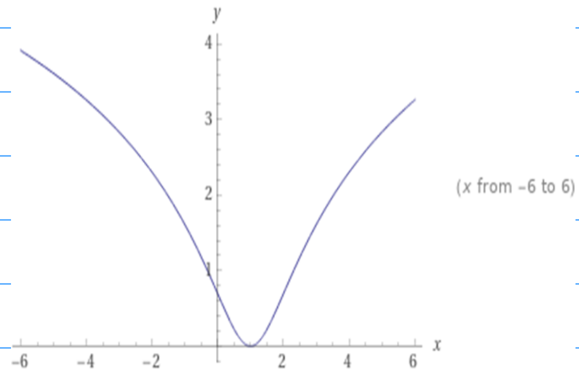
$$f''(x) = \frac{2(x^2-2x+2)-(2x-2)\cdot(2x-2)}{(x^2-2x+2)^2} = \frac{2x^2-4x+\cancel{4}-4x^2+8x-\cancel{4}}{(x^2-2x+2)^2} =$$

$$= \frac{-2x^2+4x}{(x^2-2x+2)^2}$$

$f''(x)=0 \iff -2x(x-2)=0$ $\qquad \left.\begin{array}{l} x_1 = 0 \\ x_2 = 2 \end{array}\right\}$ inflexné body

f'': $-$ | 0 | $+$ | 2 | $-$

f: $\frown$ | 0 | $\smile$ | 2 | $\frown$



f je konkávna na $(-\infty, 0\rangle$ a $\langle 2, \infty)$

konvexná $\langle 0, 2\rangle$

Pr 3. $\quad f(x) = \dfrac{x^3}{3-x^2}$ $\qquad 3 - x^2 = 0$, $\quad 3 = x^2$ $\qquad x_1 = \sqrt{3}$, $\quad x_2 = -\sqrt{3}$ $\quad \Rightarrow \quad D_f = R - \{-\sqrt{3}, \sqrt{3}\}$

$-\sqrt{3} \qquad \sqrt{3}$

$$f'(x) = \frac{3x^2(3-x^2) - x^3(-2x)}{(3-x^2)^2} = \frac{9x^2 - 3x^4 + 2x^4}{(3-x^2)^2} = \frac{x^2(9-x^2)}{(3-x^2)^2}$$

párna

$f'(x) = 0 \iff x_1 = 0, \; x_2 = 3, \; x_3 = -3$ Stac. body

f': $-$ | -3 | $+$ | $-\sqrt{3}$ | $+$ | 0 | $+$ | $\sqrt{3}$ | $+$ | 3 | $-$

f: $\searrow \quad -3 \quad \nearrow \quad -\sqrt{3} \quad \nearrow \quad 0 \quad \nearrow \quad \sqrt{3} \quad \nearrow \quad 3 \quad \searrow$

$x_3 = -3$ bod OLMIN

$x_2 = 3$ bod OLMAX

f je klesajúca na $(-\infty, -3\rangle$ a $\langle 3, \infty)$

rastúca na $\langle -3, -\sqrt{3})$, $\underbrace{(-\sqrt{3}, 0\rangle, \langle 0, \sqrt{3})}_{(-\sqrt{3}, \sqrt{3})}$, $(\sqrt{3}, 3\rangle$

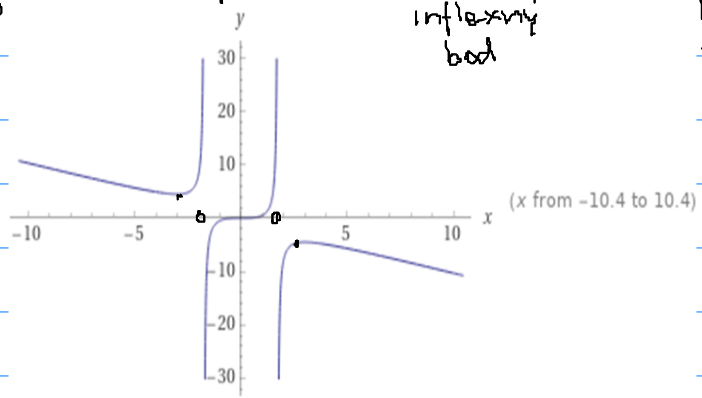
$$f''(x) = \frac{(2x(9-x^2) + x^2(-2x))\cdot(3-x^2)^{\cancel{2}} - x^2(9-x^2)\,2\,\cancel{(3-x^2)}\cdot(-2x)}{(3-x^2)^{\cancel{4}\,3}} = \frac{(18x - 2x^3 - 2x^3)(3-x^2) + 36x^3 - 4x^5}{(3-x^2)^3} = \frac{3\cdot 18x - 12x^3 - 18x^3 + \cancel{4x^5} + 36x^3 - \cancel{4x^5}}{(3-x^2)^3} =$$

$$= \frac{54x + 6x^3}{(3-x^2)^3} = \frac{6x(9+x^2)}{(3-x^2)^3}$$

$f'' = 0 \iff x = 0$ inflexný bod

f'': $+$ | $-\sqrt{3}$ | $-$ | 0 | $+$ | $\sqrt{3}$ | $-$

f: $\cup \quad -\sqrt{3} \quad \cap \quad 0 \quad \cup \quad \sqrt{3} \quad \cap$



$$f(x) = \begin{cases} \sqrt{\frac{1}{x}+1} - \sqrt{\frac{1}{x}} & x > 0 \\ 0 & x = 0 \\ \frac{\sqrt{2+x}-\sqrt{2}}{\sin x} & x < 0 \end{cases}$$

$f(0) = 0$

$$\lim_{x \to 0-} f(x) = \frac{1}{2\sqrt{2}} \neq \lim_{x \to 0+} f(x) = 0 \implies$$

$\lim_{x \to 0} f(x)$ – neexistuje $\implies$ f nie je spojitá v 0

$$\lim_{x \to 0+} \left(\sqrt{\frac{1}{x}+1} - \sqrt{\frac{1}{x}} \right) \cdot \frac{\sqrt{\frac{1}{x}+1} + \sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1} + \sqrt{\frac{1}{x}}} = \lim_{x \to 0+} \frac{\cancel{\frac{1}{x}} + 1 - \cancel{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1} + \sqrt{\frac{1}{x}}} = \lim_{x \to 0+} \frac{1}{\underbrace{\sqrt{\frac{1}{x}+1}}_{+\infty} + \underbrace{\sqrt{\frac{1}{x}}}_{+\infty}} = 0$$

$+\infty$

$$\lim_{x \to 0-} \frac{\sqrt{2+x}-\sqrt{2}}{\sin x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} = \lim_{x \to 0-} \frac{\cancel{2}+x-\cancel{2}}{\sin x} \cdot \frac{1}{\sqrt{2+x}+\sqrt{2}} = 1 \cdot \frac{1}{2\cdot\sqrt{2}}$$