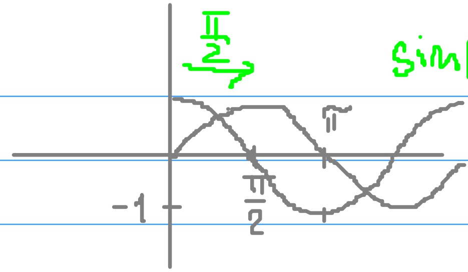
3/3,4 $\lim\limits_{x \to \pi} \frac{\sin x}{x-\pi}$

$\lim\limits_{x \to \frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}}$



$\sin\left(x+\frac{\pi}{2}\right) = \cos x$

$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta$

$$\lim_{x \to \pi} \frac{\sin x}{x-\pi} = \lim_{x \to \pi} \frac{\sin(\underbrace{x-\pi}_{\alpha} + \underbrace{\pi}_{\beta})}{x-\pi} =$$

$$= \lim_{x \to \pi} \frac{\sin(x-\pi) \cdot \overset{-1}{\overset{\shortparallel}{\cos\pi}}}{x-\pi} + \frac{\cos(x-\pi) \overset{0}{\overset{\shortparallel}{\sin\pi}}}{x-\pi} = -\lim_{x \to \pi} \frac{\sin(x-\pi)}{x-\pi} =$$

$y = x-\pi$

$x \to \pi \iff \underbrace{x-\pi}_{y} \to 0$

$$= -\lim_{y \to 0} \frac{\sin y}{y} = -1$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}} = \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(x+\frac{\pi}{2}\right)}{x-\frac{\pi}{2}} = \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(x-\frac{\pi}{2}+\pi\right)}{x-\frac{\pi}{2}} =$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(x-\frac{\pi}{2}\right) \cdot \overset{-1}{\overset{\shortparallel}{\cos\pi}} + \cos\left(x-\frac{\pi}{2}\right) \overset{0}{\overset{\shortparallel}{\sin\pi}}}{x-\frac{\pi}{2}} = \lim_{\substack{x \to \frac{\pi}{2} \\ y \to 0}} (-1) \cdot \frac{\overset{y}{\sin\left(x-\frac{\pi}{2}\right)}}{\underset{y}{x-\frac{\pi}{2}}} =$$

$= -1$

Derivácia.

1. $f(x) = 5x^3 - 3x^2 + \sqrt{x} - \frac{1}{\sqrt{x}}$

$$\sqrt{x} = x^{\frac{1}{2}}$$
$$(\sqrt{x})' = \frac{1}{2} x^{-\frac{1}{2}}$$
$$(- x^{-\frac{1}{2}})' = -\left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$$

$$f'(x) = 15x^2 - 6x + \frac{1}{2} x^{-\frac{1}{2}} + \frac{x^{-\frac{3}{2}}}{2}$$

2. $f(x) = x^2 \cos x$

$$(\cos x)' = -\sin x$$

$$f'(x) = 2x \cdot \cos x + x^2(-\sin x) = 2x\cos x - x^2 \sin x$$

3. $f(x) = \sqrt{x} \cdot e^x = x^{\frac{1}{2}} e^x$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} \cdot e^x + \sqrt{x}\, e^x$$

4. $f(x) = \frac{2x+1}{x^2+2}$

$$f'(x) = \frac{2(x^2+2) - (2x+1)2x}{(x^2+2)^2} = \frac{2x^2+4-4x^2-2x}{(x^2+2)^2} =$$

$$= \frac{-2x^2-2x+4}{(x^2+2)^2}$$

5. $f(x) = \dfrac{2x+1}{(x^2+2)^2}$

$$f'(x) = \frac{2\cdot(x^2+2)^{\cancel{2}\,1} - (2x+1)\cdot 2\,\cancel{(x^2+2)}\cdot 2x}{(x^2+2)^{\cancel{4}\,3}} =$$

$$= \frac{2x^2+4 - 8x^2 - 4x}{(x^2+2)^3} = \frac{-6x^2-4x+4}{(x^2+2)^3}$$

6. $f(x) = \dfrac{\operatorname{arctg} x}{x}$

$$f'(x) = \frac{\frac{1}{1+x^2}\cdot x - \operatorname{arctg} x}{x^2}$$

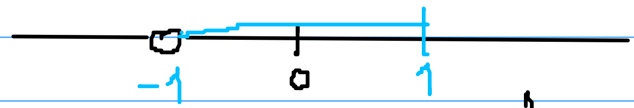
7. $f(x) = \ln(1+\sin x)$

$D_f = R - \{-\frac{\pi}{2} + 2k\pi\}$

$$f'(x) = \frac{1}{1+\sin x}\cdot \cos x$$

$D_{f'} = D_f$

8. $f(x) = \sqrt{\dfrac{1-x}{1+x}}$ $\qquad D_f = (-1, 1\rangle$

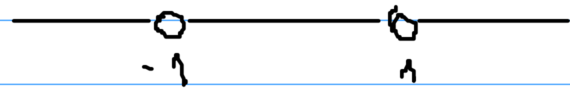


$f(x) = \left(\dfrac{1-x}{1+x}\right)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-1(1+\cancel{x}) - (1-\cancel{x})}{(1+x)^2} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\cdot\frac{-2}{(1+x)^2} =$$

$$f'(x) = -\frac{1}{(1+x)^{\frac{3}{2}}(1-x)^{\frac{1}{2}}} = -\frac{1}{\sqrt{(1+x)^3}\,\sqrt{1-x}}$$

$D_{f'} = (-1, 1)$ ✓



$f'(1)$ – neexistuje

9. $f(x) = \ln\left(x + \sqrt{x^2 + a}\right)$

$(x^2+a)' = 2x + 0$

$a \in \mathbb{R} \quad a \neq 0$

$$f'(x) = \frac{1}{x+\sqrt{x^2+a}} \cdot \left(1 + \frac{1}{2}(x^2+a)^{-\frac{1}{2}} \cdot 2x\right) = \frac{1 + \frac{x}{\sqrt{x^2+a}}}{x+\sqrt{x^2+a}} \cdot \frac{\sqrt{x^2+a}}{\sqrt{x^2+a}} =$$

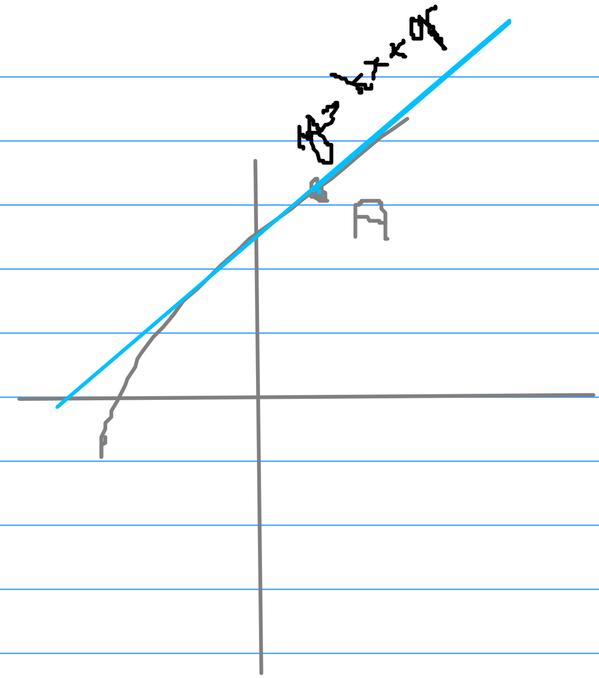
$$= \frac{\cancel{\sqrt{x^2+a} + x}}{\cancel{x+\sqrt{x^2+a}}} \cdot \frac{1}{\sqrt{x^2+a}} = \frac{1}{\sqrt{x^2+a}}$$

$$\int \frac{1}{\sqrt{x^2+3}}\,dx = \ln\left(x + \sqrt{x^2+3}\right) + C$$

Rovnica dotyčnice

10. $f(x)=\sqrt{x^2+5}$ $A=[2,3]$

Dotyčnica ku G_f v bode A



$f(2) \stackrel{?}{=} 3$ $f(2)=\sqrt{4+5}=3$ ✓

$k=f'(2)$

$f'(x)=\frac{1}{2}(x^2+5)^{-\frac{1}{2}}\cdot 2x=\frac{x}{\sqrt{x^2+5}}$ $f'(2)=\frac{2}{\sqrt{4+5}}=\frac{2}{3}$

$y-3=\frac{2}{3}(x-2)$

$df(x,2)=\frac{2}{3}(x-2)$

11. $f(x) = e^{\frac{1}{1+x^2}} = e^{(1+x^2)^{-1}}$ $\qquad A = [?, \sqrt{e}]$

$\sqrt{e} = e^{\frac{1}{1+x^2}}$

$e^{\frac{1}{2}} = e^{\frac{1}{1+x^2}} \iff \frac{1}{2} = \frac{1}{1+x^2} \iff 2 = 1 + x^2 \iff x^2 = 1 \iff x_1 = 1$ alebo $x_2 = -1$

Buď $A_1 = [1, \sqrt{e}]$ alebo $A_2 = [-1, \sqrt{e}]$

$f'(x) = (-1) e^{\frac{1}{1+x^2}} \cdot (1+x^2)^{-2} \cdot 2x =$ $\qquad [e^x]' = e^x \qquad (1+x^2)^{-1}$

$= - e^{\frac{1}{1+x^2}} \cdot \frac{2x}{(1+x^2)^2}$

$f'(1) = - e^{\frac{1}{2}} \cdot \frac{2}{4} = - \frac{1}{2}\sqrt{e}$ $\qquad f'(-1) = - e^{\frac{1}{2}} \cdot \frac{-2}{4} = \frac{1}{2}\sqrt{e}$

v bode A_1 $\qquad$ v bode A_2

$y - \sqrt{e} = -\frac{1}{2}\sqrt{e}\,(x-1)$ $\qquad y - \sqrt{e} = \frac{1}{2}\sqrt{e}\,(x+1)$

12. $f(x) = \dfrac{1}{x^2 + \frac{5}{2}x - 1} = \left(x^2 + \frac{5}{2}x - 1\right)^{-1}$ $\qquad A = [2, 2]$

$$2 = \frac{1}{x^2 + \frac{5}{2}x - 1}$$

$$2x^2 + 5x - 2 = 1$$
$$2x^2 + 5x - 3 = 0$$

$$x_{1,2} = \frac{-5 \pm \sqrt{25 + 4 \cdot 2 \cdot 3}}{4} = -\frac{5}{4} \pm \frac{7}{4} = \begin{cases} \frac{2}{4} = \frac{1}{2} \\ -\frac{12}{4} = -3 \end{cases}$$

$A_1 = \left[\frac{1}{2}, 2\right]$ $\qquad A_2 = [-3, 2]$

$$f'(x) = -\left(x^2 + \frac{5}{2}x - 1\right)^{-2} \cdot \left(2x + \frac{5}{2}\right) = -\frac{2x + \frac{5}{2}}{\left(x^2 + \frac{5}{2}x - 1\right)^2}$$

A_1: $f'\left(\frac{1}{2}\right) = -\dfrac{1 + \frac{5}{2}}{\left(\frac{1}{4} + \frac{5}{4} - 1\right)^2} = -\dfrac{\frac{7}{2}}{\left(\frac{1}{2}\right)^2} = -14$ $\qquad y - 2 = -14 \cdot \left(x - \frac{1}{2}\right)$

$df\left(x, \frac{1}{2}\right) = -14\left(x - \frac{1}{2}\right)$

A_2: $f'(-3) = -\dfrac{-6 + \frac{5}{2}}{\left(9 - \frac{15}{2} - 1\right)^2} = -\dfrac{-\frac{7}{2}}{\left(\frac{1}{2}\right)^2} = 14$ $\qquad y - 2 = 14(x + 3)$

$df(x, -3) = 14(x + 3)$