$$f(x)=\frac{\ln(x^2+3)}{x^2+5} \qquad f'(x)=\frac{\frac{1}{x^2+3}\cdot 2x\cdot(x^2+5)-\ln(x^2+3)\cdot 2x}{(x^2+5)^2}=2x\,\frac{(x^2+5)-\ln(x^2+3)(x^2+3)}{(x^2+3)(x^2+5)^2}$$

$$\frac{2x}{(x^2+3)(x^2+5)}-\frac{2x\ln(x^2+3)}{(x^2+5)^2}$$

Pr1. $f(x)=3x^2+2x+\sqrt{x}-\frac{1}{\sqrt{x}}=3x^2+2x+x^{\frac{1}{2}}-x^{-\frac{1}{2}}$ $\qquad (x^n)'=nx^{n-1}$

$f'(x)=6x+2+\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$ $\qquad (f\cdot g)'=f'g+fg'$

Pr 2. $f(x)=x\cdot\cos x$

$f'(x)=1\cos x+x(-\sin x)=\cos x-x\sin x$

Pr3. $f(x)=\sqrt{x}\cdot e^x \qquad f'(x)=\frac{1}{2}x^{-\frac{1}{2}}\cdot e^x+\sqrt{x}\cdot e^x$

Pr.4 $f(x)=\frac{x-3}{x^2+3} \qquad f'(x)=\frac{1\cdot(x^2+3)-(x-3)\cdot 2x}{(x^2+3)^2}=\frac{x^2+3-2x^2+6x}{(x^2+3)^2}=\frac{-x^2+6x+3}{(x^2+3)^2}$

Pr 4b

$$f(x) = \frac{x-3}{(x^2+3)^2}$$

$$f'(x) = \frac{1\,(x^2+3)^2 - (x-3)\cdot 2(x^2+3)\cdot 2x}{(x^2+3)^4} = \frac{\cancel{(x^2+3)}\left[(x^2+3) - 4x(x-3)\right]}{\cancel{(x^2+3)}\cdot(x^2+3)^3}$$

$$= \frac{-3x^2+12x+3}{(x^2+3)^3}$$

Pr 5

$$f(x) = \frac{\operatorname{arctg} x}{x}$$

$$f'(x) = \frac{\frac{1}{1+x^2}\cdot x - \operatorname{arctg} x \cdot 1}{x^2} = \frac{x - (1+x^2)\operatorname{arctg} x}{(1+x^2)\,x^2}$$

Pr 6

$$f(x) = \ln(1+\sin x)$$

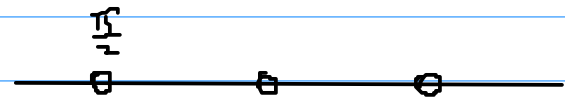
$$g(f(x)) = g'(f(x))\cdot f'(x)$$

$$f'(x) = \frac{1}{1+\sin x}\cdot \cos x$$

$$1+\sin x > 0$$

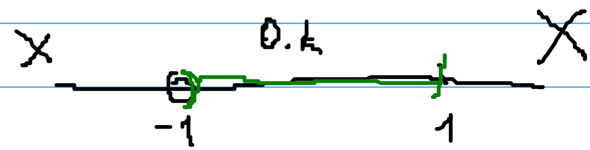
$$1 > \sin x$$

$$D_f = R - \left\{\frac{\pi}{2} + 2k\pi\right\}$$



Pr 7

$$f(x) = \sqrt{\frac{1-x}{1+x}} \qquad D_f = (-1, 1\rangle$$



$$f(x) = \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \frac{-1(1+x) - (1-x)}{(1+x)^2} = \frac{1}{\cancel{2}}\,\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{-\cancel{2}}{(1+x)^2} =$$

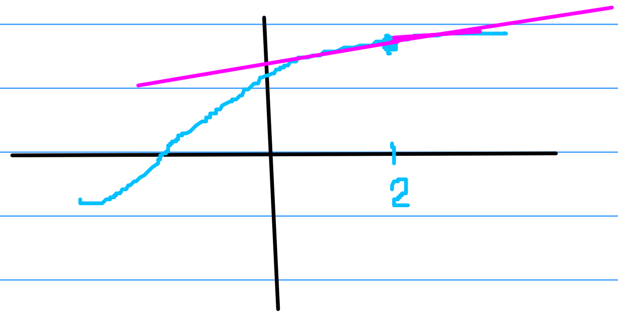
$$= \frac{-1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}$$

Pr 8 $\quad f(x) = \ln\left(x + \sqrt{x^2+a}\right)$

$$f'(x) = \frac{1}{x+\sqrt{x^2+a}} \cdot \left(1 + \frac{1}{\cancel{2}} \frac{\cancel{2}x}{\sqrt{x^2+a}}\right) = \frac{1}{\cancel{x+\sqrt{x^2+a}}} \cdot \frac{\cancel{\sqrt{x^2+a}+x}}{\sqrt{x^2+a}} = \frac{1}{\sqrt{x^2+a}}$$

Dotyčnica

Pr 9. $\quad f(x) = \sqrt{x^2+5} \qquad A = [2, ?]$



$$y_0 = f(2) = \sqrt{4+5} = 3 \qquad A = [\underset{x_0}{2}, 3]$$

$$y - y_0 = k(x - x_0)$$

$$k = f'(2)$$

$$f'(x) = \frac{1}{2}(x^2+5)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+5}}$$

$$f'(2) = \frac{2}{3}$$

$$y - 3 = \frac{2}{3}(x - 2)$$

$f(x) = e^{\frac{1}{1+x^2}}$ $\quad$ $A = [?, \sqrt{e}]$

$e^{\frac{1}{1+x^2}} = \sqrt{e} = e^{\frac{1}{2}} \quad \Big| \ln$

$\frac{1}{1+x^2} = \frac{1}{2}$

$1+x^2 = 2$

$x^2 = 1$

$x_1 = 1$ alebo $x_2 = -1$

$A_1 = [1, \sqrt{e}]$

$A_2 = [-1, \sqrt{e}]$

$\frac{1}{1+x^2} = (1+x^2)^{-1}$

$f'(x) = e^{\frac{1}{1+x^2}} \cdot -1 \cdot (1+x^2)^{-2} \cdot 2x =$

$= e^{\frac{1}{1+x^2}} \cdot \frac{-2x}{(1+x^2)^2}$

Pre $x_1 = 1$

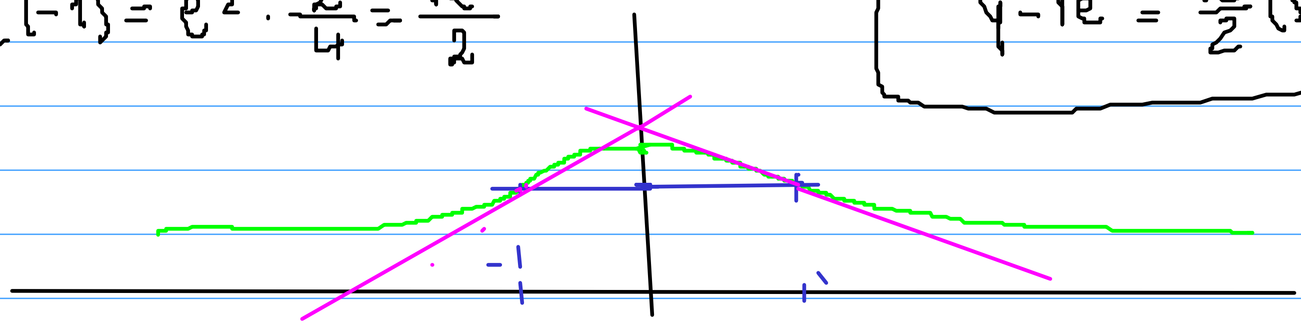
$f'(1) = e^{\frac{1}{2}} \cdot \frac{-2}{4} = -\frac{\sqrt{e}}{2}$

Pre $x_2 = -1$

$f'(-1) = e^{\frac{1}{2}} \cdot \frac{2}{4} = \frac{\sqrt{e}}{2}$

Dotyčnica

$$\boxed{\begin{aligned} y - \sqrt{e} &= -\frac{\sqrt{e}}{2}(x-1) \\ y - \sqrt{e} &= \frac{\sqrt{e}}{2}(x+1) \end{aligned}}$$



Pr 10 $\quad f(x)=\dfrac{1}{x^2+\frac{5}{2}x-1}= \qquad A=[?,2] \qquad =\left(x^2+\frac{5}{2}x-1\right)^{-1}$

$$\frac{1}{x^2+\frac{5}{2}x-1}=2$$

$$\frac{1}{2}=x^2+\frac{5}{2}x-1$$

$$0=x^2+\frac{5}{2}x-\frac{3}{2}$$

$$\downarrow$$

$$\left(\frac{5}{2}\right)^2=\frac{25}{4}$$

$$x_{1,2}=\frac{-\frac{5}{2}\pm\sqrt{\frac{25}{4}+4\cdot\frac{3}{2}}}{2}=\frac{-\frac{5}{2}\pm\sqrt{\frac{25+24}{4}}}{2}=\frac{-\frac{5}{2}\pm\frac{7}{2}}{2}$$

$$x_{1,2}=\begin{cases}\frac{1}{2}\\ -3\end{cases}$$

$A_1=\left[\frac{1}{2},2\right] \qquad A_2=[-3,2]$

$$f'(x)=-1\left(x^2+\frac{5}{2}x-1\right)^{-2}\cdot\left(2x+\frac{5}{2}\right)=\frac{2x+\frac{5}{2}}{\left(x^2+\frac{5}{2}x-1\right)^2}$$

Pre $x=\frac{1}{2}$

$$f'\left(\frac{1}{2}\right)=\frac{\frac{7}{2}}{\left(\frac{1}{4}+\frac{5}{4}-1\right)^2}=\frac{7}{2}\cdot\frac{1}{1/4}=14$$

$x=-3$

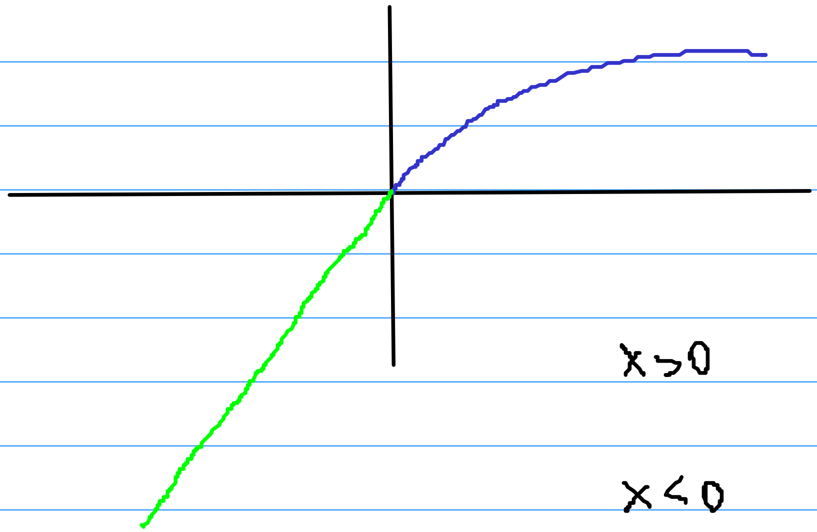
$$f'(-3)=\frac{-\frac{7}{2}}{\left(9-\frac{15}{2}-1\right)^2}=\frac{-\frac{7}{2}}{1/4}=-14$$

$$\boxed{\begin{aligned}(y-2)&=14\left(x-\frac{1}{2}\right)\\ (y-2)&=-14(x+3)\end{aligned}}$$

Pr 11.

$$f(x) = \begin{cases} \ln(1+x) & x > 0 \\ ax + b & x \leq 0 \end{cases}$$

Zvoľme a, b tak, aby f bola diferencovateľná v bode 0



$$\lim_{x \to 0+} \ln(1+x) = \lim_{x \to 0-} ax+b$$

$$\boxed{0 = b}$$

$x > 0$ $\quad f'(x) = \left[\ln(1+x)\right]' = \frac{1}{1+x} \cdot 1$

$x < 0$ $\quad f'(x) = a$

$$\lim_{x \to 0+} f'(x) = \lim_{x \to 0+} \frac{1}{1+x} = 1 = \lim_{x \to 0-} f'(x) = a$$

$$\boxed{a = 1 \qquad b = 0}$$