Cvičeni 3 – Limita

1. $$\lim_{x\to\infty}\frac{2x^2+5x-3}{5x^3-2x^2+2}=\lim_{x\to\infty}\frac{\cancel{x^2}\left(2+\underset{\to 0}{\frac{5}{x}}-\underset{\to 0}{\frac{3}{x^2}}\right)}{x^{\cancel{3}}\left(5-\underset{\to 0}{\frac{2}{x}}+\underset{\to 0}{\frac{2}{x^3}}\right)}=\lim_{x\to\infty}\frac{1}{x}\cdot\frac{2}{5}=0$$

$$\lim_{x\to-\infty}\frac{2x^2+5x-3}{5x^3-2x^2+2}=\lim_{x\to-\infty}\frac{1}{x}\cdot\frac{2}{5}=0$$

2. $$\lim_{x\to\infty}\frac{x^4+3x^3-5x+1}{5x^3-2x^2+x-7}=\lim_{x\to\infty}\frac{x^{\cancel{4}\,1}\left(1+\frac{3}{x}-\frac{5}{x^3}+\frac{1}{x^4}\right)}{\cancel{x^3}\left(5-\frac{2}{x}+\frac{1}{x^2}-\frac{7}{x^3}\right)}=\lim_{x\to\infty}x\cdot\frac{1}{5}=+\infty$$

$$\lim_{x\to-\infty}\frac{x^4+3x^3-5x+1}{5x^3-2x^2+x-7}=\text{—}\,||\,\text{—}=\lim_{x\to-\infty}x\cdot\frac{1}{5}=-\infty$$

3. $$\lim_{x\to+\infty}\frac{2x^3-4x+5}{5x^3+3x^2-2}=\lim_{x\to+\infty}\frac{\cancel{x^3}\left(2-\frac{4}{x^2}+\frac{5}{x^3}\right)}{\cancel{x^3}\left(5+\frac{3}{x}-\frac{2}{x^3}\right)}=\frac{2}{5}$$

$$\lim_{x\to-\infty}\frac{2x^3-4x+5}{5x^3+3x^2-2}=\frac{2}{5}$$

I. $\frac{2}{3}$ II. $+\infty$ III.

Pr 3b

$$\lim_{x\to+\infty}\frac{\sqrt{2x^4-3x^3+7x-1}}{\sqrt[3]{3x^6-5x^5+2x^2-3}}=\lim_{x\to+\infty}\frac{\sqrt{x^4\left(2-\frac{3}{x}+\frac{7}{x^3}-\frac{1}{x^4}\right)}}{\sqrt[3]{x^6\left(3-\frac{5}{x}+\frac{2}{x^4}-\frac{3}{x^6}\right)}}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^2}\cdot\sqrt{2-\frac{3}{x}+\frac{7}{x^3}-\frac{1}{x^4}}}{\cancel{x^2}\cdot\sqrt[3]{3-\frac{5}{x}+\frac{2}{x^4}-\frac{3}{x^6}}}=\frac{\sqrt{2}}{\sqrt[3]{3}}$$

0 0

Pr 4. Typ „$(+\infty)-(+\infty)$"

$$\lim_{x\to\infty}\frac{x^3}{x^2+1}-x=\lim_{x\to\infty}\frac{\cancel{x^3}-(\cancel{x^3}+x)}{x^2+1}=\lim_{x\to\infty}\frac{-x}{x^2+1}=\underline{\underline{0}}$$

5

$$\lim_{x\to\infty}\frac{x^4}{x^2+1}-(x^2+3)=\lim_{x\to\infty}\frac{x^4-(x^2+3)(x^2+1)}{x^2+1}=\lim_{x\to\infty}\frac{\cancel{x^4}-(\cancel{x^4}+4x^2+3)}{x^2+1}=$$

$$=\lim_{x\to\infty}\frac{-4x^2-3}{1x^2+1}=\underline{\underline{-4}}$$

$\cancel{\frac{\infty}{\infty}}$

$-\infty$

6. $\lim_{x\to\infty} \frac{x^4}{x^2+1} - (x^2+2x+3) = \lim_{x\to\infty} \frac{x^4-(x^2+2x+3)\cdot(x^2+1)}{x^2+1} =$

$= \lim_{x\to\infty} \frac{\cancel{x^4} - (\cancel{x^4}+x^2+2x^3+2x+3x^2+3)}{x^2+1} = \lim_{x\to\infty} \frac{-2x^3-4x^2-2x-3}{x^2+1} = -\infty$

$=//$

7 $\lim_{x\to\infty} \sqrt{x^2+1} - \sqrt{x^2-1} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{1} \cdot \frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}} =$

$= \lim_{x\to\infty} \frac{\cancel{x^2}+1-(\cancel{x^2}-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}} = 0$

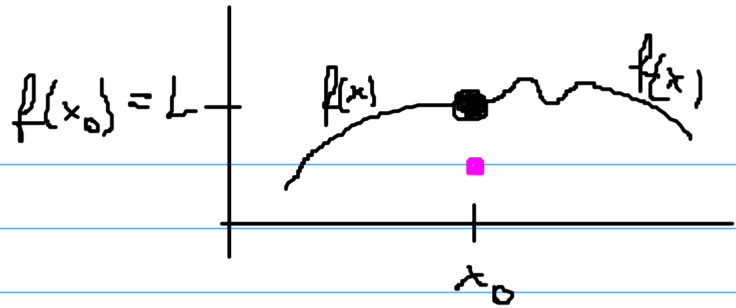
8. $\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x+1} \cdot \frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}} = \lim_{x\to\infty} \frac{\cancel{x^2}+x+\cancel{1}-(\cancel{x^2}-x+\cancel{1})}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}} =$

1

2 $\lim_{x\to\infty} \frac{2x}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}} = \lim_{x\to\infty} \frac{2\cdot\cancel{x}}{\cancel{x}\cdot\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\cancel{x}\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}} = \frac{2}{1+1} = 1$

0 0

Spojitosť



9.

$$f(x) = \begin{cases} x \sin \frac{1}{x^2} & \text{ak } x \neq 0 \\ 0 & \text{ak } x = 0 \end{cases}$$

$$\lim_{x \to 0} \underbrace{x}_{} \underbrace{\sin \frac{1}{x^2}}_{0} = 0 = f(0) \quad \Rightarrow \quad \text{funkcia } f \text{ je spojitá v bode } 0.$$

$\lim\limits_{x \to 0} x = 0$ $\qquad -1 \leq \sin \frac{1}{x^2} \leq 1$

ohr.

10.

$$f(x) = \begin{cases} x \sin \frac{1}{x^2} & \text{ak } x \neq 0 \\ 2 & \text{ak } x = 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = 0 \neq 2 = f(0) \quad \Rightarrow \quad f \text{ nie je spojitá v bode } x_0 = 0$$

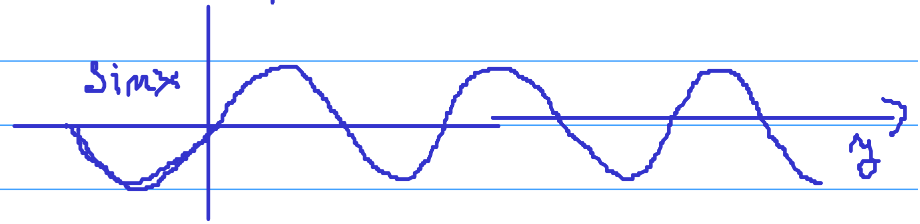
11.

11.

$$f(x) = \begin{cases} \sin \frac{1}{x^2} & \text{ak } x \neq 0 \\ 0 & \text{ak } x = 0 \end{cases} .$$

$\lim\limits_{x \to 0} \sin \frac{1}{x^2}$ – neexistuje $\Rightarrow$ f nie je spojitá v bode $x_0 = 0$.

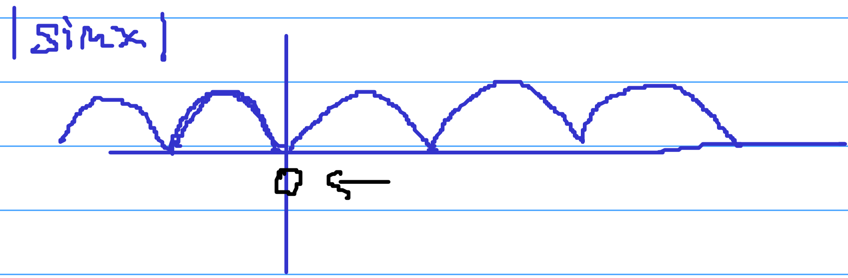
$\lim\limits_{x \to 0} \underbrace{\frac{1}{x^2}}_{=y} = +\infty$ $\qquad \lim\limits_{y \to +\infty} \sin y$ – neexistuje



12.

$$f(x) = \begin{cases} \frac{|\sin x|}{x} & \text{ak } x \neq 0 \\ 1 & \text{ak } x = 0 \end{cases} .$$

$$\lim_{x \to 0} \frac{|\sin x|}{x}$$



$$\lim_{x \to 0+} \frac{|\sin x|}{x} = \lim_{x \to 0+} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0-} \frac{|\sin x|}{x} = \lim_{x\to 0-}\frac{-\sin x}{x} = -1 \qquad L^+ = 1 \neq -1 = L^- \Rightarrow$$

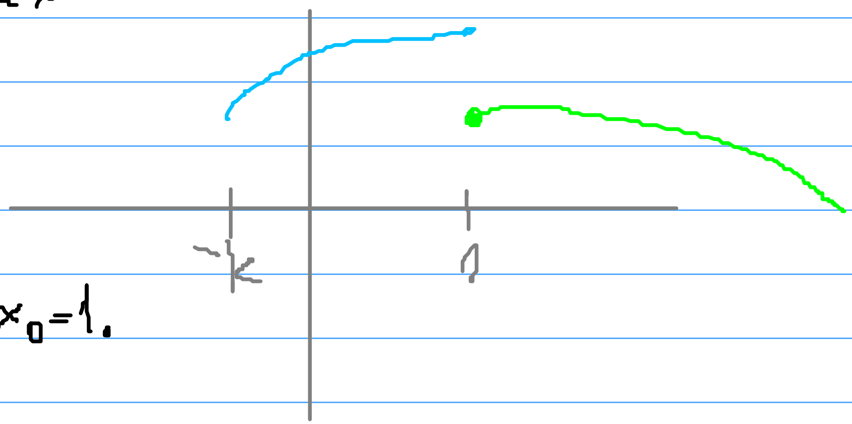
$$\lim_{x\to 0}\frac{|\sin x|}{x} \text{ — neex.} \Rightarrow$$

funkcia nie je spojitá v bode $x_0=0$

13. $$f(x) = \begin{cases} \sqrt{x+k} & \text{pre } -k\le x\le 1 \\ \dfrac{\sqrt{x}-1}{x^2-1} & \text{pre } 1<x\end{cases}$$

$-k<1$

$k>-1$



Nájdite k tak, aby funkcia f bola spojitá v bode $x_0=1$.

- $$\lim_{x\to 1-}\sqrt{x+k} = \sqrt{1+k}$$

- $$\lim_{x\to1+}\frac{\sqrt x -1}{x^2-1} = \lim_{x\to1+}\frac{\sqrt x-1}{(x-1)}\cdot\frac{\sqrt x+1}{\sqrt x+1}\cdot\frac{1}{(x+1)}$$

$$= \lim_{x\to1+}\frac{\cancel{x-1}}{\cancel{x-1}}\cdot\frac{1}{\sqrt x+1}\cdot\frac{1}{x+1} = \frac12\cdot\frac12=\frac14$$

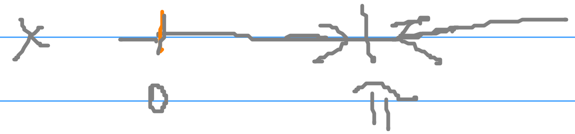
$$L^- = \sqrt{1+k} = \frac{1}{4} = L^+$$

$$1+k = \frac{1}{16}$$

$$k = \frac{1}{16} - 1 = -\frac{15}{16}$$

f je spojitá v bode $x_0 = 1$ pre $k = -\frac{15}{16}$.

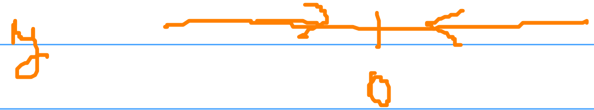
$$\lim_{x \to \pi} \frac{\sin x}{x - \pi} =$$



$x = y + \pi$

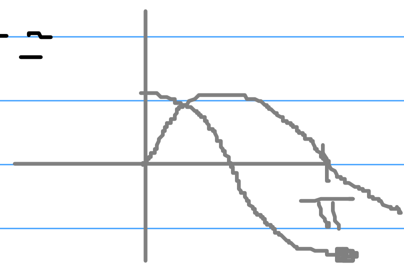
$x - \pi = y$

$$= \lim_{y \to 0} \frac{\sin(y + \pi)}{y} =$$



$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$$= \lim_{y \to 0} \frac{\sin y \overset{-1}{\cos \pi} + \cos y \overset{0}{\sin \pi}}{y} = \lim_{y \to 0} \frac{-\sin y}{y} =$$



$$= -1$$