$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}\cdot\frac{x^{\frac{2}{3}}+x^{\frac{1}{3}}2+2^2}{x^{\frac{2}{3}}+x^{\frac{1}{3}}\cdot 2+2^2}=$$

$$=\lim_{x\to 8}\frac{\cancel{x-8}}{\cancel{(x-8)}\cdot\left(x^{\frac{2}{3}}+2x^{\frac{1}{3}}+4\right)}=\frac{1}{4+4+4}=\frac{1}{12}$$

Cvičenie 3. Nevlastné limity

Pr 1

$$\lim_{x\to\infty}\frac{2x^2+3x-1}{5x^3-2x^2+3}=\lim_{x\to\infty}\frac{\cancel{x^2}\left(2+3\frac{1}{x}-\frac{1}{x^2}\right)}{x^{\cancel{3}\,1}\left(5-2\frac{1}{x}+\frac{3}{x^3}\right)}=\lim_{x\to\infty}\frac{1}{x}\,\frac{2}{5}=0$$

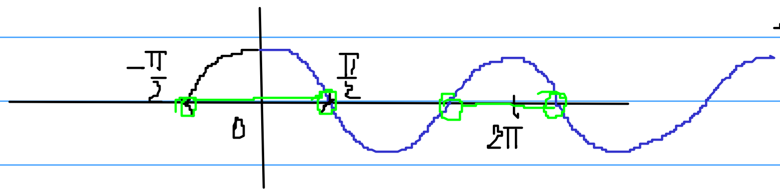
$x\to-\infty$ $\qquad$ $x\to-\infty$ $\qquad$ 0

$0 \quad 0 \quad 0 \quad 0$

Pr $\qquad f(x)=\ln(\cos x)\qquad D_f=?\qquad \ln y\qquad y>0$

$y=\cos x>0$

$$D_f=\bigcup_{k\in\mathbb{Z}}\left(-\frac{\pi}{2}+2k\pi,\ \frac{\pi}{2}+2k\pi\right)$$



Pr2

$$\lim_{x\to\infty}\frac{2x^4+3x^2-1}{5x^3-2x^2+3}=\lim_{x\to\infty}\frac{x^{\cancel{4}\,1}\left(2+\frac{3}{x^2}-\frac{1}{x^4}\right)}{\cancel{x^3}\left(5-\frac{2}{x}+\frac{3}{x^3}\right)}=\lim_{x\to\infty}x\cdot\frac{2}{5}=+\infty$$

$x\to-\infty$ $\qquad$ $x\to-\infty$ $\qquad =-\infty$

Pr 3. $\lim\limits_{x\to\infty} \dfrac{2x^3+3x-1}{5x^3-2x^2+3} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^3}\left(2+\frac{3}{x^2}-\frac{1}{x^3}\right)}{\cancel{x^3}\left(5-\frac{2}{x}+\frac{3}{x^3}\right)} = \dfrac{2}{5}$

$x\to-\infty$ $\qquad = \dfrac{2}{5}$

Limita typu „$\infty-\infty$“

Pr 4. $\lim\limits_{x\to\infty} \dfrac{x^3}{x^2+1} - x = \lim\limits_{x\to\infty} \dfrac{\cancel{x^3}-\cancel{x^3}-x^1}{x^2+1} = 0$

Pr 5. $\lim\limits_{x\to\infty} \dfrac{x^4}{x^2+1} - x^2+3 = \lim\limits_{x\to\infty} \dfrac{\cancel{x^4}-\cancel{x^4}+3x^2-x^2+3}{x^2+1} = \dfrac{2}{1} = 2$

Pr 6. $\lim\limits_{x\to\infty} \dfrac{x^4}{x^2+1} - x^2+3x+1 = \lim\limits_{x\to\infty} \dfrac{\cancel{x^4}-\cancel{x^4}+3x^3+\cancel{x^2}-\cancel{x^2}+3x+1}{x^2+1} = +\infty$

$x\to-\infty$ $\qquad = -\infty$

Pr 7. $\lim\limits_{x\to\infty} \sqrt{x^2+1}-\sqrt{x^2-1} \cdot \dfrac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}+1-(\cancel{x^2}-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim\limits_{x\to\infty} \dfrac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}} = 0$

Pr 8. $\lim\limits_{x\to\infty} \sqrt{x^2+x+1}-\sqrt{x^2-x+1} \cdot \dfrac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}+x+\cancel{1}-(\cancel{x^2}-x+\cancel{1})}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}} = \lim\limits_{x\to\infty} \dfrac{2x}{\text{ditto}}$

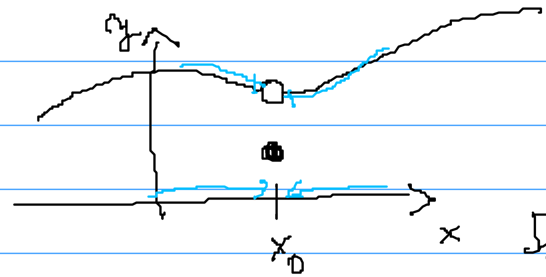
$x\to-\infty$

$$= \lim_{x \to \infty} \frac{2x}{\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}+\sqrt{x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)}} = \lim_{x\to\infty} \frac{2x}{x+x} = 1$$

$x \to -\infty$

$$= \lim_{x\to -\infty} \frac{2x}{|x|+|x|} = \lim_{x\to-\infty} \frac{\cancel{2}x}{\cancel{2}|x|} = -1$$

Spojitosť



$D_f = R$

$$\lim_{x\to x_0} f(x) = f(x_0)$$

Pr 9.
$$f(x) = \begin{cases} x \cdot \sin\frac{1}{x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{x\to 0} x \cdot \sin\frac{1}{x^2} = 0 = f(0) \Rightarrow f \text{ je spojitá v bode } x_0 = 0.$$

0 ohraničená

$$-1 \leq \sin\frac{1}{x^2} \leq 1$$

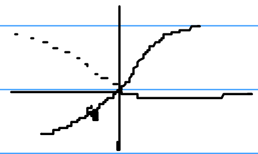
Pr 9b
$$f(x) = \begin{cases} x \cdot \sin\frac{1}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$\lim_{x\to 0} x \cdot \sin\frac{1}{x^2} = 0 \neq 1 = f(x_0) \Rightarrow f \text{ nie je spojitá v bode } x_0 = 0.$$

$x_0 = 0$

Pr 10. $f(x) = \begin{cases} \dfrac{|\sin x|}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ $\quad x_0 = 0$

$$\lim_{x \to 0} \frac{|\sin x|}{x}$$

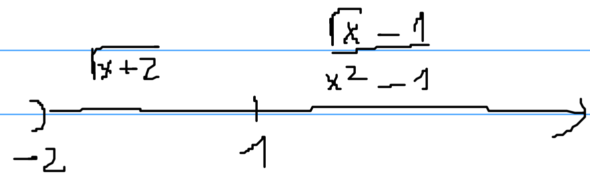


$$\lim_{x \to 0+} \frac{\sin x}{x} = 1$$

$$\neq \Rightarrow \lim_{x \to 0} \frac{|\sin x|}{x} - \text{neex.} \Rightarrow f \text{ nie je spojitá v } x_0 = 0.$$

$$\lim_{x \to 0-} \frac{-\sin x}{x} = -1$$

Pr 11. $f(x) = \begin{cases} \sqrt{x+2} & -2 \leq x \leq 1 \\ \dfrac{\sqrt{x} - 1}{x^2 - 1} & 1 < x \end{cases}$ $\quad x_0 = 1$



$$\lim_{x \to 1-} \sqrt{x+2} = \sqrt{3}$$

$$\lim_{x \to 1+} \frac{\sqrt{x} - 1}{x^2 - 1} = \lim_{x \to 1+} \frac{(\sqrt{x} - 1)(\sqrt{x} + 1)}{(x-1)(x+1) \cdot (\sqrt{x} + 1)} = \lim_{x \to 1+} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)(\sqrt{x} + 1)} = \frac{1}{4}$$

$$\sqrt{3} \neq \frac{1}{4} \Rightarrow f \text{ nie je spojitá v bode } x_0 = 1$$

Pr 11 b

$$f(x) = \begin{cases} \sqrt{x+k} & -2 \le x \le 1 \\ \dfrac{\sqrt{x}-1}{x^2-1} & 1 < x \end{cases}$$

$k = ?$ aby f bola spojitá?

$$\lim_{x \to 1-} \sqrt{x+k} = \frac{1}{4}$$

$$1 + k = \frac{1}{16}$$

$$k = -\frac{15}{16}$$