Cvičenie 2 – Limity

1. $$\lim_{x\to 2} \frac{x^2+2x+3}{x^2-1} = \frac{4+4+3}{4-1} = \frac{11}{3}$$

2. $$\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{\cancel{x-2}}{(\cancel{x-2})(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}$$

3. $$\lim_{x\to 2} \frac{x^2+x-6}{x^2-4} = \lim_{x\to 2} \frac{(\cancel{x-2})(x+3)}{(\cancel{x-2})(x+2)} = \frac{5}{4}$$

3.1 $$\lim_{x\to 2} \frac{x^2+x-5}{x^2-4} = \lim_{x\to 2} \frac{\overset{\to 1}{x^2+x-5}}{\underset{\to 0}{(x-2)}\underset{\to 4}{(x+2)}} \text{ – neexistuje}$$

$$\left.\begin{array}{l} \displaystyle\lim_{x\to 2+} \underset{\to +\infty}{\frac{1}{x-2}} \cdot \underset{\to \frac{1}{4}}{\frac{x^2+x-5}{x+2}} = +\infty \\[2em] \displaystyle\lim_{x\to 2-} \underset{\to -\infty}{\frac{1}{x-2}} \cdot \underset{\to \frac{1}{4}}{\frac{x^2+x-5}{x+2}} = -\infty \end{array}\right\} \Rightarrow \text{limita neexistuje}$$

4. $\lim\limits_{x\to 5} \dfrac{\overset{\nearrow 0}{\sqrt{x-1}-2}}{\underset{\searrow 0}{\sqrt{2x-1}-3}} = \lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-2}{\sqrt{2x-1}-3} \cdot \dfrac{\sqrt{2x-1}+3}{\sqrt{2x-1}+3} = \lim\limits_{x\to 5} \dfrac{(\sqrt{x-1}-2)\cdot(\sqrt{2x-1}+3)}{(2x-1)-9} =$

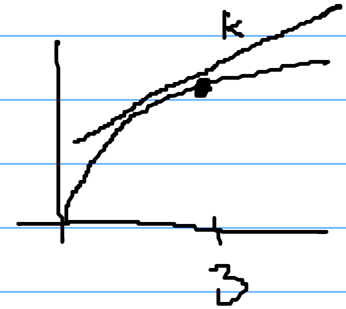
$(a-b)\cdot(a+b)$

$\lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-2}{2x-10} \cdot \dfrac{\sqrt{x-1}+2}{\sqrt{x-1}+2} \cdot \dfrac{(\sqrt{2x-1}+3)}{1} = \lim\limits_{x\to 5} \dfrac{(x-1)-4}{2(x-5)} \cdot \dfrac{\sqrt{2x-1}+3}{\sqrt{x-1}+2} =$

$= \lim\limits_{x\to 5} \dfrac{\cancel{x-5}}{2(\cancel{x-5})} \cdot \dfrac{\sqrt{2x-1}+3}{\sqrt{x-1}+2} = \dfrac{6}{2\cdot 4} = \dfrac{3}{2\cdot 2} = \dfrac{3}{4}$

5. $\lim\limits_{x\to 3} \dfrac{\sqrt{x}-\sqrt{3}}{x-3} = \lim\limits_{x\to 3} \dfrac{\sqrt{x}-\sqrt{3}}{x-3} \cdot \dfrac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}} = \lim\limits_{x\to 3} \dfrac{\cancel{x-3}}{\cancel{x-3}} \cdot \dfrac{1}{\sqrt{x}+\sqrt{3}} = \dfrac{1}{2\sqrt{3}}$

$f(x)=\sqrt{x}$ $\quad x_0 = 3$ $\quad \lim\limits_{x\to x_0} \dfrac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$

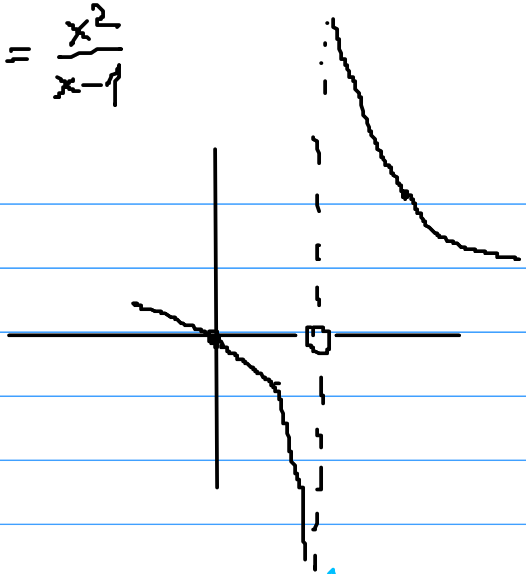


6. $\lim\limits_{x\to 1+} \dfrac{\overset{\nearrow 1}{x^2}}{\underset{\searrow 0}{x-1}} = +\infty$ $\qquad$ $\lim\limits_{x\to 1-} \dfrac{\overset{\nearrow 1}{x^2}}{\underset{\searrow 0}{x-1}} = -\infty$

$x-1>0$ $\qquad$ $x-1<0$

$\Rightarrow \lim\limits_{x\to 1} \dfrac{x^2}{x-1}$ – neexistuj

$f(x)=\frac{x^2}{x-1}$



Pr 7

$\lim_{x\to 1+} \frac{x^2}{(x-1)^2} = +\infty$ $\qquad x^2 \to 1$

$(x-1)^2 \to 0$ $\qquad \to 0$

$\lim_{x\to 1-} \frac{x^2}{(x-1)^2} = +\infty$ $\qquad x^2 \to 1$

$(x-1)^2 \to 0$

Pr 8

$\lim_{x\to 0+} e^{\frac{1}{x}} = +\infty \quad \neq \quad \lim_{x\to 0-} e^{\frac{1}{x}} = 0$

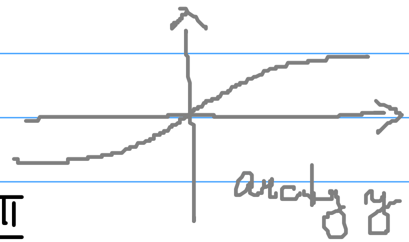
$y=\frac{1}{x} \to +\infty$

$y=\frac{1}{x} \to -\infty$

$-0.1 \quad -0.01 \quad -0.0001$

$\frac{1}{-0.0001} = -\frac{1}{1/10000} = -10^4$

$\lim_{x\to 0} e^{\frac{1}{x}}$ – neexistuje



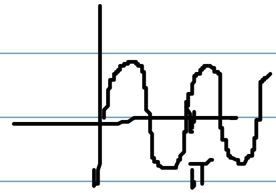
Pr 9.

$\lim_{x\to 0+} \operatorname{arctg}\frac{1}{x} = \frac{\pi}{2}$

$\lim_{x\to 0-} \operatorname{arctg}\frac{1}{x} = -\frac{\pi}{2}$

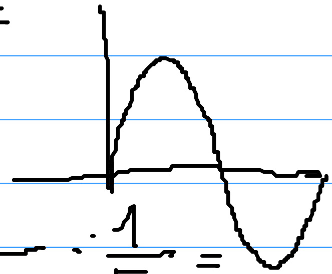
$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}$$

$\sin 2x \neq 2\sin x$



$\sin 2x$

Pr 10 $$\lim_{x\to 0}\frac{\sin 2x}{5x}=\lim_{x\to 0}\underbrace{\frac{\sin 2x}{2x}}_{\to 1}\cdot\frac{2\cancel{x}}{5\cancel{x}}=\frac{2}{5}$$



$2\sin x$

Pr 11 $$\lim_{x\to 0}\frac{\operatorname{tg}3x}{5x}=\lim_{x\to 0}\underbrace{\frac{\sin 3x}{3x}}_{\to 1}\ \frac{3\cancel{x}}{\cos 3x}\cdot\frac{1}{5\cancel{x}}=$$

$$=1\cdot\frac{3}{1}\cdot\frac{1}{5}=\frac{3}{5}$$

Pr 12
$$\lim_{x\to 0}\frac{\cos x-1}{x^2}=\lim_{x\to 0}\frac{\cos x-1}{x^2}\cdot\frac{\cos x+1}{\cos x+1}=\lim_{x\to 0}\frac{\cos^2 x-1}{x^2}\cdot\frac{1}{\cos x+1}=$$

$$=\lim_{x\to 0}-\frac{\sin^2 x}{x^2}\cdot\frac{1}{\cos x+1}=-\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\cdot\frac{1}{\cos x+1}=$$

$1-\cos^2 x=\sin^2 x$ $$=-1\cdot\frac{1}{2}=-\frac{1}{2}$$

Pr 13. $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{\cos 3x - 1} \cdot \dfrac{\cos 3x + 1}{\cos 3x + 1} \cdot \dfrac{\cos x + 1}{\cos x + 1} = \lim\limits_{x\to 0} \dfrac{\cos^2 x - 1}{\cos^2 3x - 1} \cdot \dfrac{\cos 3x + 1}{\cos x + 1} =$

$= \lim\limits_{x\to 0} \dfrac{-\sin^2 x}{x^2} \cdot \dfrac{(3x)^2}{-\sin^2 3x} \cdot \dfrac{\cos 3x + 1}{\cos x + 1} \cdot \dfrac{1}{9}$

$= \lim\limits_{x\to 0} -1 \left(\dfrac{\sin x}{x}\right)^2 \cdot (-1) \left(\dfrac{3x}{\sin 3x}\right)^2 \cdot \dfrac{\cos 3x + 1}{\cos x + 1} \cdot \dfrac{1}{9} = 1 \cdot 1 \cdot 1 \cdot \dfrac{2}{2} \cdot \dfrac{1}{9} = \dfrac{1}{9}$

Pr 14 $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{\sqrt{x^2+x+1} - \sqrt{x+1}} \cdot \dfrac{\sqrt{x^2+x+1} + \sqrt{x+1}}{\sqrt{x^2+x+1} + \sqrt{x+1}} \cdot \dfrac{1 + \cos x}{1 + \cos x} =$

$= \lim\limits_{x\to 0} \dfrac{1 - \cos^2 x}{(x^2+x+1) - (x+1)} \cdot \dfrac{\sqrt{x^2+x+1} + \sqrt{x+1}}{1 + \cos x} = \lim\limits_{x\to 0} \dfrac{\sin^2 x}{x^2} \cdot \dfrac{\sqrt{x^2+x+1} + \sqrt{x+1}}{1 + \cos x} =$

$= 1 \cdot \dfrac{2}{2} = 1$

Pr 15.

$$\lim_{x \to 0} \underbrace{x}_{\to 0} \cdot \underbrace{\operatorname{arctg}}_{\text{ohraničená}} \left(\frac{\sqrt{2x+1}}{\sin \frac{1}{x}} \right) = 0$$

$$-\frac{\pi}{2} < \operatorname{arctg} \left(\frac{\sqrt{2x+1}}{\sin \frac{1}{x}} \right) < \frac{\pi}{2}$$

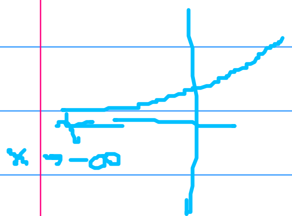
Pr 16

$$\lim_{x \to \infty} \frac{\sin x}{x} = \lim_{x \to \infty} \underbrace{\frac{1}{x}}_{\to 0} \cdot \underbrace{\sin x}_{\text{ohraničená}} = 0 = \lim_{x \to -\infty} \frac{\sin x}{x}$$

$$-1 \leq \sin x \leq 1$$

Pr 17

$$\lim_{x \to \infty} \underbrace{e^{-x}}_{\to 0} \underbrace{\left(2\sin x + \cos x \right)}_{\text{ohraničená}} = 0$$



$$-3 \leq 2\sin x + \cos x \leq 3$$