Pr1. $\lim\limits_{x\to 2} \dfrac{x^2+1}{x^2-1} = \dfrac{5}{3}$

Pr2. $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4} = \lim\limits_{x\to 2} \dfrac{\cancel{x-2}}{\cancel{(x-2)}\cdot(x+2)} = \dfrac{1}{4}$

Pr3 $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x^2-4} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(x+2)} = \dfrac{5}{4}$

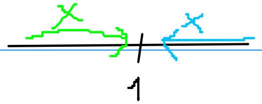
Pr4. $\lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-2}{\sqrt{2x-1}-3} = \lim\limits_{x\to 5} \dfrac{(\sqrt{x-1}-2)\cdot(\sqrt{2x-1}+3)\,(\sqrt{x-1}+2)}{(\sqrt{2x-1}-3)\cdot(\sqrt{2x-1}+3)\,(\sqrt{x-1}+2)} = \lim\limits_{x\to 5} \dfrac{\overset{\overset{1}{\cancel{x-5}}}{(x-1-4)}\cdot\overset{6}{(\sqrt{2x-1}+3)}}{\underset{\underset{2}{\cancel{2x-10}}}{(2x-1-9)}\cdot\underset{4}{(\sqrt{x-1}+2)}} =$

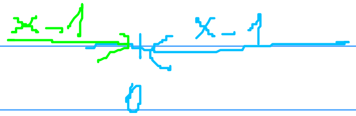
$= \dfrac{1}{2}\cdot\dfrac{6}{4} = \dfrac{3}{4}$

Pr5. $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}\cdot\dfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} = \lim\limits_{x\to 0} \dfrac{2+x-2}{x} = 1$

Pr6. $\lim\limits_{x\to 2} \dfrac{x^3-2x^2-x+2}{x^2-x-2} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}\cdot x^2 - \overset{1}{\cancel{(x-2)}}}{\cancel{(x-2)}(x+1)} = \lim\limits_{x\to 2} \dfrac{x^2-1}{x+1} = \dfrac{3}{3} = 1$

Pr 7. $\lim\limits_{x \to 1+} \dfrac{\overset{\nearrow 1}{x^2}}{\underset{\searrow 0^+}{x-1}} = +\infty$





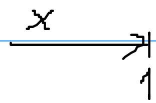
Pr 8. $\lim\limits_{x \to 1-} \dfrac{\overset{\nearrow 1}{x^2}}{\underset{\searrow 0^-}{x-1}} = -\infty$

$\lim\limits_{x \to 1} \dfrac{x^2}{x-1}$ — neex.

Pr 9. $\lim\limits_{x \to 1+} \dfrac{x^2}{\sqrt{x-1}} = +\infty$

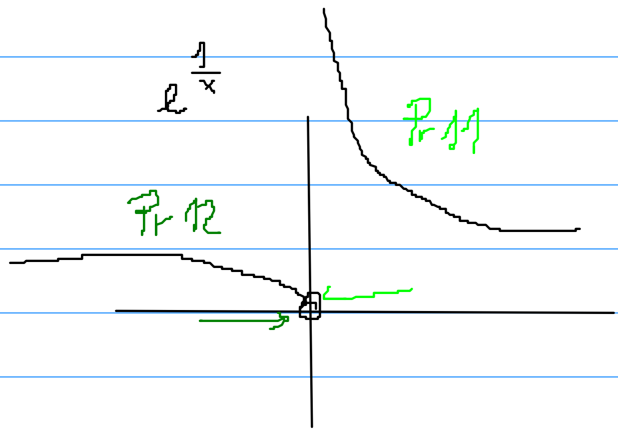
Pr 10. $\lim\limits_{x \to 1-} \dfrac{x^2}{\sqrt{x-1}} = \cancel{-\infty}\ \cancel{+\infty}$

Nezmyselný



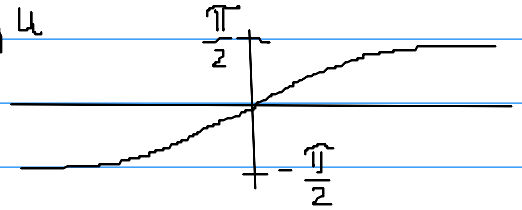
Pr 11 $\lim\limits_{x \to 0+} e^{\frac{1}{x}} = +\infty$

Pr 12 $\lim\limits_{x \to 0-} e^{\frac{1}{x}} = 0$

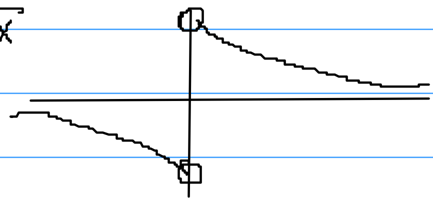


Pr 13. $\lim\limits_{x\to 0+} \operatorname{arctg}\frac{1}{x} = \frac{\pi}{2}$ $\qquad$ $\lim\limits_{x\to 0-} \operatorname{arctg}\frac{1}{x} = -\frac{\pi}{2}$

$\operatorname{arctg} u$



$\operatorname{arctg}\frac{1}{x}$



Pr. 14 $\lim\limits_{x\to 0}\frac{\sin 2x}{3x} = \lim\limits_{x\to 0}\underbrace{\frac{\sin 2x}{2x}}_{1}\cdot\frac{2x}{3x} = \frac{2}{3}$

$\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

Pr 15 $\lim\limits_{x\to 0}\frac{\operatorname{tg} 3x}{5x} = \lim\limits_{x\to 0}\underbrace{\frac{\sin 3x}{5x}}_{\frac{3}{5}}\underbrace{\frac{1}{\cos 3x}}_{1} = \frac{3}{5}$

$\left(\frac{x}{\sin x}\right)^2 = \frac{1}{\left(\frac{\sin x}{x}\right)^2}$

$\|$

Pr 16. $\lim\limits_{x\to 0}\frac{x^2}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x} = \lim\limits_{x\to 0}\frac{x^2\cdot(1+\cos x)}{1-\cos^2 x} = \lim\limits_{x\to 0}\underbrace{\frac{x^2}{\sin^2 x}}_{=1}\cdot\frac{1+\cos x}{1} = \frac{2}{1} = 2$

Pr 17 $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{\cos 3x - 1} \cdot \dfrac{(\cos 3x+1)(\cos x+1)}{(\cos 3x+1)(\cos x+1)} = \lim\limits_{x\to 0} \dfrac{\cos^2 x - 1}{\cos^2 3x - 1} \cdot \dfrac{\cos 3x + 1}{\cos x + 1} = \lim\limits_{x\to 0} \dfrac{-\sin^2 x}{x^2} \cdot \dfrac{x^2}{-\sin^2 3x} \cdot \dfrac{\cos 3x+1}{\cos x+1}$

$= -1 \cdot -\dfrac{1}{9} \cdot 1 = \dfrac{1}{9}$

$= \dfrac{9x^2}{9\sin^2 3x} = \dfrac{1}{9}\left(\dfrac{3x}{\sin 3x}\right)^2$

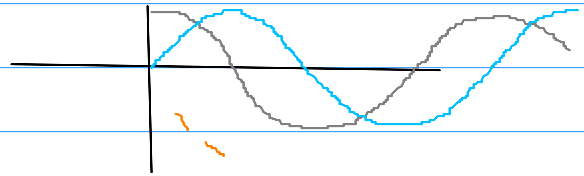
Pr 18 $\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{x+2}-\sqrt{2}} \cdot \dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} = \lim\limits_{x\to 0} \dfrac{\sin x}{x+\cancel{2}-\cancel{2}} \cdot \dfrac{\sqrt{x+2}+\sqrt{2}}{1} = 1 \cdot 2\sqrt{2}$

Pr 18b $\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{x+2}-2} = \dfrac{0}{\sqrt{2}-2} = 0$

Pr 19 $\lim\limits_{x\to \infty} \dfrac{\sin x}{x} = \lim\limits_{x\to\infty} \underbrace{\dfrac{1}{x}}_{\downarrow\, 0} \cdot \sin x = 0$

ohraničená

$\lim\limits_{x\to\infty} \dfrac{\cos x}{1}$ — neexistuje



Pr 20 $\lim\limits_{x\to\infty} \underbrace{e^{-x}}_{\downarrow\, 0} (3\cos x - 2\sin 5x) = 0$

ohraničená

$-5 \le 3\cos x - 2\sin 5x \le 5$