Definičné obory.

Pr 1. $f(x) = \frac{1}{x^2-5x+4}$

$x^2-5x+4=0$

$(x-1)\cdot(x-4)=0$

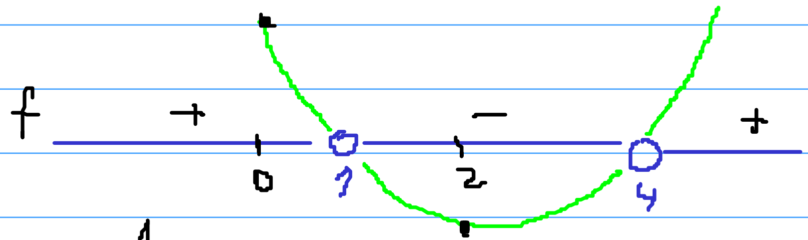
$x_1=1 \quad x_2=4$

$D_f = R - \{1,4\}$

Pr 2. $f(x) = \frac{x-2}{x^2-5x+4}$

$D_f = R - \{1,4\}$

Pr 3. $f(x) = \sqrt{\frac{1}{x^2-5x+4}}$



$\frac{1}{x^2-5x+4} > 0 \iff x^2-5x+4 > 0$

pre $x=0$ $\quad 0^2-5\cdot 0+4 = 4 > 0$

pre $x=2$ $\quad 4-10+4 = -2 < 0$

$D_f = (-\infty, 1) \cup (4, \infty)$

Pr 4. $f(x) = \sqrt{\frac{x-2}{x^2-5x+4}}$

$\frac{x-2}{x^2-5x+4} \geq 0 \iff$

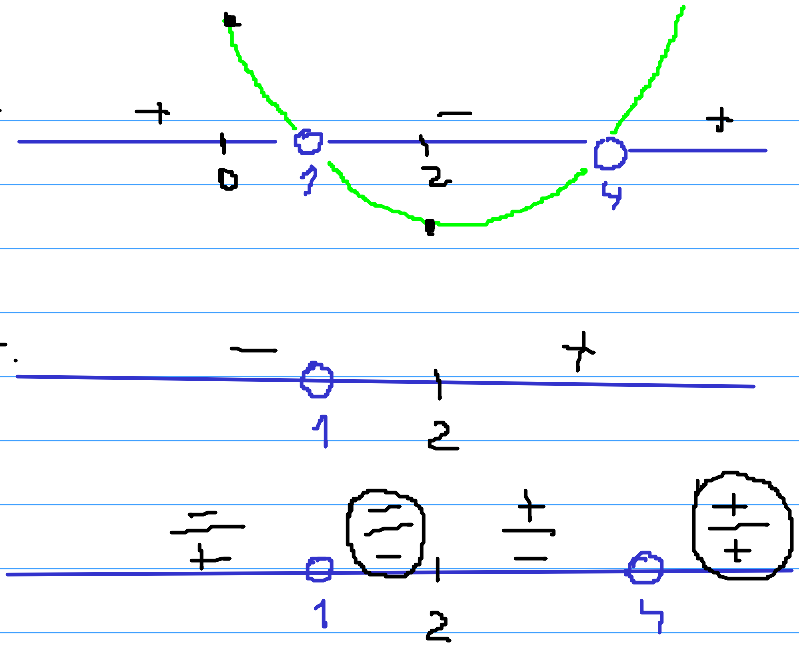
alebo

I $\quad x-2 \geq 0 \quad \wedge \quad x^2-5x+4 > 0$

II $\quad x-2 \leq 0 \quad \wedge \quad x^2-5x+4 < 0$

Men.

Čit.

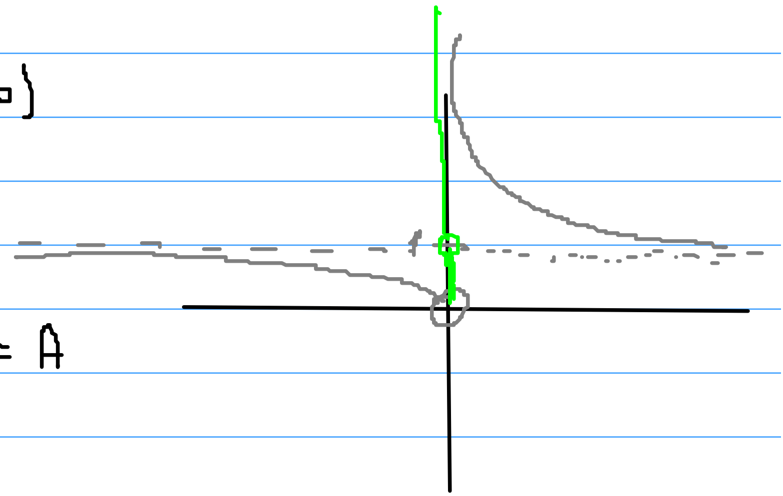


$$D_f = (1,2\rangle \cup (4,\infty)$$

Pr5. $\quad f(x) = e^{\frac{1}{x}}$ $\qquad D_f = R - \{0\} = (-\infty,0) \cup (0,\infty)$

$y = e^{\frac{1}{x}}$ $\qquad y > 0$

$\ln y = \ln e^{\frac{1}{x}}$ $\qquad \ln e^{A} = A$



$\ln y = \frac{1}{x}$

$x = \frac{1}{\ln y}$ $\qquad f^{-1}(y) = \frac{1}{\ln y}$ $\qquad D_{f^{-1}} = (0,1) \cup (1,\infty) = H_f$

$1 \notin D_f$

$\ln 1 = 0$

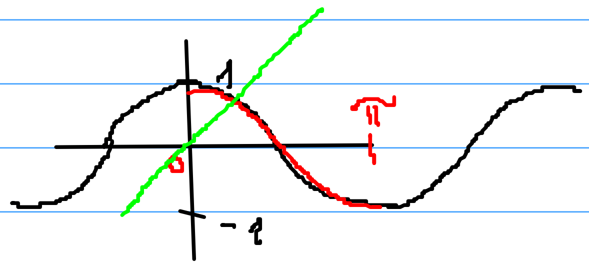
Pr 6. $f(x) = \arccos \frac{1}{x}$ $\qquad D_f =$

$R - \{0\}$,

$\langle -1, 1 \rangle$.

$R - \langle -1, 1 \rangle = (-\infty, -1) \cup (1, \infty)$

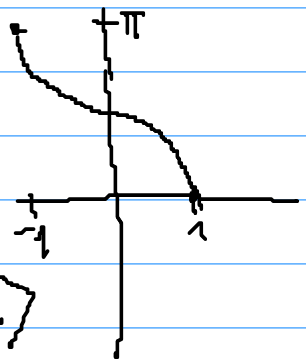
cos x



$y = \cos x$

$x = \arccos y$

arccos y



$D_{\arccos} = \langle -1, 1 \rangle$

$-1 \le \frac{1}{x} \le 1$

Buď $\boxed{x > 0}$ $\qquad \frac{1}{x} \le 1 \quad \Big| \cdot x \qquad \wedge \qquad -1 \le \frac{1}{x}$

$1 \le x$

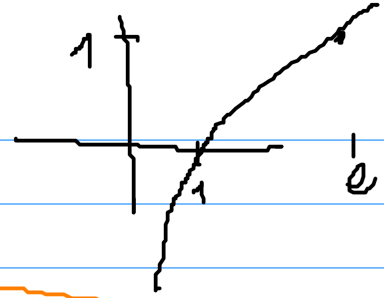
Alebo $\boxed{x < 0}$ $\qquad -1 \le \frac{1}{x} \quad \Big| \cdot x$

$-x \ge 1 \quad \Big| \cdot (-1)$

$x \le -1$

$D_f = (-\infty, -1\rangle \cup \langle 1, \infty)$

Pr 7. $f(x) = \sqrt{1 - \ln(x-1)}$



$\ln(x-1)$ je def pre $x > 1$

$1 - \ln(x-1) \geq 0$ $\qquad \ln e = 1 \Rightarrow x - 1 = e$

$x = e + 1$

$D_f = (1, e+1\rangle$

$f^{-1} = ?$ $\qquad y = \sqrt{1 - \ln(x-1)} \quad \Big|^2$ $\quad$ Neekvivalentná úprava $\quad y \geq 0$!!

$y^2 = 1 - \ln(x-1) \quad | -1$

$y^2 - 1 = -\ln(x-1) \quad | \cdot(-1)$

$1 - y^2 = \ln(x-1) \quad | \exp$

$e^{1-y^2} = x - 1 \quad | +1$

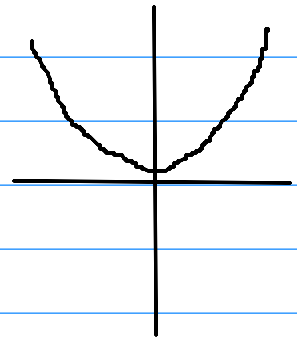
$x = \boxed{1 + e^{1-y^2} = f^{-1}(y)}$

$D_{f^{-1}} = R_0^+ = H_f = \langle 0, \infty)$

Parita

Př 8 $f(x) = x^2$

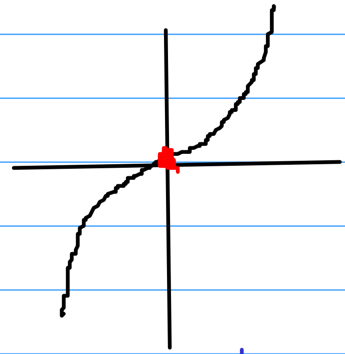
$f(-x) = f(x)$



Párna

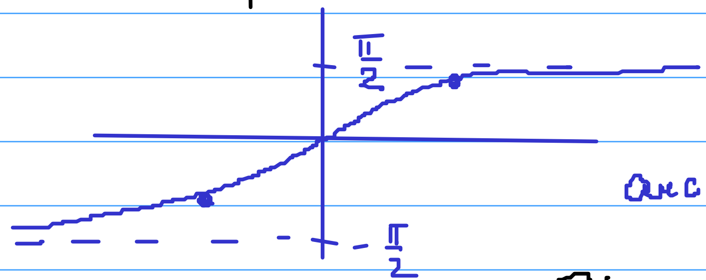
Př 9. $f(x) = x^3$

$f(-x) = -f(x)$



Nepárna

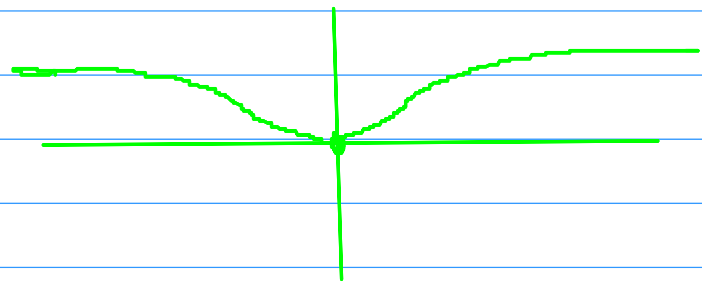
Př 10 $f(x) = \operatorname{arctg} x^2$



$\operatorname{arctg} z$

$\operatorname{arctg}(-z) = -\operatorname{arctg} z$

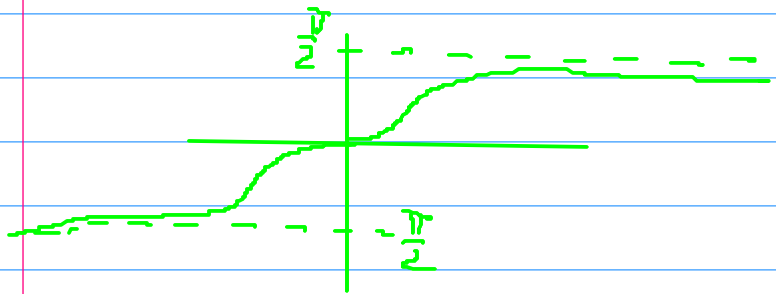
$f(-x) = \operatorname{arctg}(-x)^2 = \operatorname{arctg} x^2 = f(x)$ Párna



$\operatorname{arctg}(x^2)$

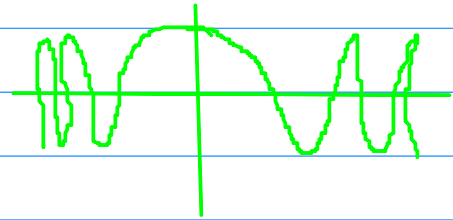
Pr 11 $f(x) = \operatorname{arctg}(x^3)$ Nepárna

$$f(-x) = \operatorname{arctg}((-x)^3) = \operatorname{arctg}(-x^3) = -\operatorname{arctg} x^3 = -f(x) \Rightarrow$$



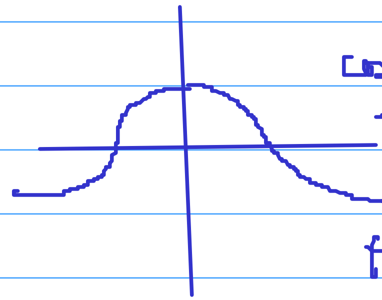
Pr 12 $f(x) = \cos(x^2)$ Párna

$$f(-x) = \cos(-x)^2 = \cos x^2 = f(x)$$



Pr 13 $f(x) = \cos(x^3)$ Párna.

$$\underline{\underline{f(-x)}} = \cos(-x)^3 = \cos(-x^3) = \cos x^3 = \underline{\underline{f(x)}}$$



cosinus je párna funkcia

Pr 14 $f(x) = e^{(x^2)}$

$D_f = R$ Párna ||

ANI – ANI |

Pr 15 $f(x) = e^{(x^3)}$

$D_f = R$

Nepárna

ANI – ANI ||

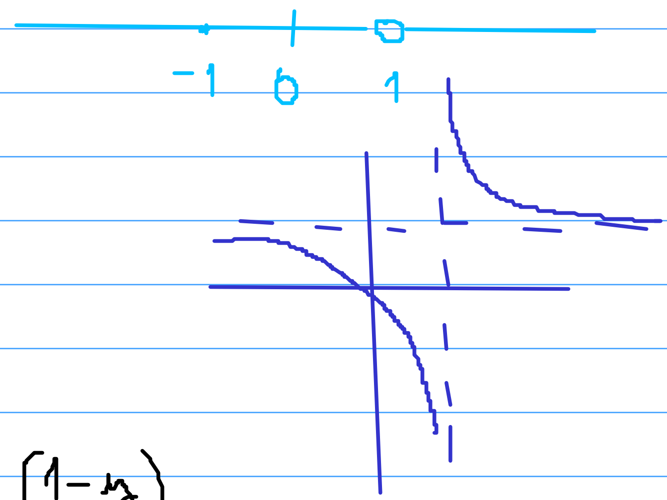
Pr 14 $f(-x) = e^{(-x)^2} = e^{x^2} = f(x) =$ Párna

Pr 15 $f(1) = e^{1^3} = e \doteq 2{,}7$ $\quad f(1) \neq f(-1) \Rightarrow$ nie je párna

$f(-1) = e^{(-1)^3} = e^{-1} = \frac{1}{e} = \frac{1}{2{,}7} \doteq 0{,}3\ldots$ $\quad f(1) \neq -f(-1) \Rightarrow$ nie je nepárna

ANI Párna, ANI Nepárna

Pr 16. $f(x) = \frac{x+2}{x-1}$ $\qquad D_f = R - \{1\}$



$y = \frac{x+2}{x-1} \quad \Big| \cdot (x-1)$

$yx - y = x + 2 \quad \Big| -2$

$yx - y - 2 = x \quad \Big| -xy$

$-y - 2 = x - xy = x(1-y) \quad \Big| (1-y)$

$\frac{y+2}{y-1} = \frac{-y-2}{1-y} = x$ $\qquad f^{-1}(y) = \frac{y+2}{y-1}$ $\qquad f^{-1}(x) = \frac{x+2}{x-1}$ $\qquad D_{f^{-1}} = R - \{1\}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \shortparallel$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad H_f$

$x \geq -1$

Pr 17 $\quad f(x) = e^{\sqrt{x^3+1}} \qquad D_f = \langle -1, \infty)$

$$y = e^{\sqrt{x^3+1}}$$

$$\ln y = \ln\left(e^{\sqrt{x^3+1}}\right) = \sqrt{x^3+1} \quad \Big|^2$$

Pozor $\ln y \geq 0$

$$(\ln y)^2 = x^3 + 1$$

$y \geq 1$

$$\sqrt[3]{(\ln y)^2 - 1} = x$$

$$f^{-1}(y) = \sqrt[3]{(\ln y)^2 - 1} \qquad D_{f^{-1}} = \langle 1, \infty) = H_f$$

$$\left[(\ln y)^2 - 1\right]^{\frac{7}{2}}$$