Ćwiczenie 12

Pr1

$$\int_{e}^{3} \frac{1}{x \ln x}\, dx = \int_{1}^{\ln 3} \frac{1}{y}\, dy = \Big[\ln|y|\Big]_{1}^{\ln 3} = \ln(\ln 3) - \ln 1 = \ln(\ln 3)$$

Subst. $y = \ln x$ $\quad x = e \quad \ln e = 1 = y$

$dy = \frac{1}{x} dx \quad x = 3 \quad \ln 3 = y$

Pr2.

$$\int_{-1}^{1} \frac{x}{\sqrt{5-4x}}\, dx = \frac{1}{-4} \int_{-1}^{1} \frac{x}{\sqrt{5-4x}} \cdot (-4)\, dx = -\frac{1}{4} \int_{9}^{1} \frac{\frac{5-y}{4}}{\sqrt{y}}\, dy = \frac{1}{16} \int_{1}^{9} \frac{5-y}{\sqrt{y}}\, dy =$$

Subst. $y = 5-4x \quad 4x = 5-y \quad x = \frac{5-y}{4}$

$dy = -4dx$

$x = -1 \longrightarrow y = 9$

$x = 1 \longrightarrow y = 1$

$$= \frac{1}{16} \int_{1}^{9} 5y^{-\frac{1}{2}} - y^{\frac{1}{2}}\, dy = \frac{1}{16}\left(\Big[5 y^{\frac{1}{2}} \cdot 2\Big]_{1}^{9} - \Big[\frac{2 y^{\frac{3}{2}}}{3}\Big]_{1}^{9} \right) = \frac{1}{16}\left(30 - 10 - \frac{2}{3}(27-1)\right) = \frac{20 - \frac{2}{3}\cdot 26}{16}$$

Pr 3 $$\int_0^1 \frac{x+2}{x^2+2x+5}\,dx = \frac{1}{2}\int_0^1 \frac{2x+4}{x^2+2x+5}\,dx = \frac{1}{2}\int_0^1 \frac{2x+2}{x^2+2x+5} + \frac{2}{x^2+2x+5}\,dx =$$

$x^2+2x+5=0$ $\quad D = 4-4.5 = -16 < 0$

$2x+2$

$$= \frac{1}{2}\Big[\ln|x^2+2x+5|\Big]_0^1 + \int_0^1 \frac{1}{x^2+2x+1+4}\,dx =$$

$\int \frac{1}{y^2+a^2}\,dy = \frac{1}{a}\,\mathrm{arctg}\,\frac{y}{a}$

$$= \frac{1}{2}\Big[\ln|x^2+2x+5|\Big]_0^1 + \int_0^1 \frac{1}{(x+1)^2+2^2}\,dx =$$

$\int \frac{1}{(x+1)^2+2^2}\,dx = \frac{1}{2}\,\mathrm{arctg}\,\frac{x+1}{2}$

$$= \frac{1}{2}\Big[\ln|x^2+2x+5|\Big]_0^1 + \frac{1}{2}\Big[\mathrm{arctg}\,\frac{x+1}{2}\Big]_0^1 =$$

$$= \frac{1}{2}(\ln 8 - \ln 5) + \frac{1}{2}\left(\mathrm{arctg}\,1 - \mathrm{arctg}\,\frac{1}{2}\right) = \frac{1}{2}\left(\ln\frac{8}{5} + \frac{\pi}{4} - \mathrm{arctg}\,\frac{1}{2}\right)$$

Pr 4. $$\int_0^1 \frac{x+2}{x^2+3x+2}\,dx = \int_0^1 \frac{\cancel{x+2}}{(x+1)\cancel{(x+2)}}\,dx = \int_0^1 \frac{1}{x+1}\,dx = \Big[\ln|x+1|\Big]_0^1 = \ln 2 - \ln 1 = \underline{\underline{\ln 2}}$$

$x^2+3x+2=0$ $\quad D = 9-4.1.2 = 1 > 0$

$(x+1)(x+2)=0$

$$\frac{x+2}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$$

$x=-1 \quad 1 = A.1 \quad A=1$

$x+2 = A(x+2) + B(x+1) \qquad x=-2 \quad 0 = B(-1) \quad B=0$

Pr 5. $\displaystyle\int_1^{64} \frac{\sqrt{x}-1}{(1+\sqrt[3]{x})\,x}\,dx = \int_1^{2} \frac{t^3-1}{(1+t^2)\cdot \cancel{t^6}}\,6\cancel{t^5}\,dt = 6\int_1^2 \frac{t^3-1}{t^3+t}\,dt = 6\int_1^2 1-\frac{t+1}{t^3+t}\,dt =$

Subst. $x=t^6$ $\quad t=x^{\frac{1}{6}}$ $\quad 1^{\frac{1}{6}}=1$

$dx=6t^5dt$ $\quad 64^{\frac{1}{6}}=2$

$$t^3-1 : t^3+t = 1 - \frac{t+1}{t^3+t}$$
$$-(t^3+t)$$
$$-t-1$$

$$= 6\left[t\right]_1^2 - 6\int_1^2 \frac{t+1}{t\cdot(t^2+1)}\,dt = 6\cdot 1 - 6\int_1^2 \frac{1}{t}-\frac{t-1}{t^2+1}\,dt = 6-6\left[\ln|t|\right]_1^2 + 6\int_1^2 \frac{t-1}{t^2+1}\,dt =$$

$$\frac{t+1}{t(t^2+1)} = \frac{A}{t}+\frac{Bt+C}{t^2+1} = \qquad t+1 = A(t^2+1)+(Bt+C)\cdot t$$

$t=0 \quad 1=A \qquad\qquad A=1$

$$= \frac{1}{t}-\frac{t}{t^2+1}$$

$$1 = At^2+Bt^2+Ct+A$$

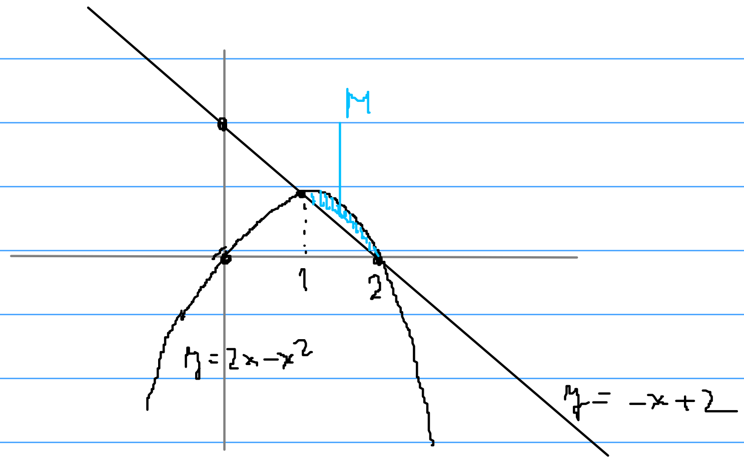
$t^2 \quad 0=A+B \quad \Rightarrow B=-1$

$t \quad 1=C \quad C=1$

$$=6-6\ln 2+3\int_1^2 \frac{2t}{t^2+1}\,dt - 6\int_1^2 \frac{1}{t^2+1}\,dt = 6-6\ln 2+3\left[\ln|t^2+1|\right]_1^2 - 6\left[\operatorname{arctg} t\right]_1^2 =$$

$$= 6-6\ln 2+3\ln 5-3\ln 2-6\operatorname{arctg} 2+6\operatorname{arctg} 1$$

Pr 6. Vypočítajme obsah oblasti M ohraničenej čiarami $y = 2x - x^2$, $y = -x + 2$



$$\int_a^b f(x) - g(x)\, dx = \int_1^2 2x - x^2 - (-x+2)\, dx =$$

$$= \int_1^2 -x^2 + 3x - 2\, dx = \left[-\frac{x^3}{3} + 3\frac{x^2}{2} - 2x \right]_1^2 =$$

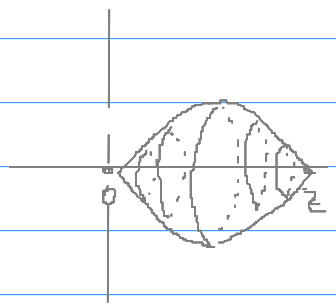
$$= -\frac{8}{3} + 6 - 4 - \left(-\frac{1}{3} + \frac{3}{2} - 2 \right) =$$

$$= -\frac{7}{3} + 4 - \frac{3}{2} = \frac{-14 + 24 - 9}{6} = \frac{1}{6}$$

$$2x - x^2 = -x + 2$$

$$0 = x^2 - 3x + 2 \qquad x_1 = 1 \qquad x_2 = 2$$

Pr7 Objem rotačného telesa. Rotácia M okolo osi x $\qquad M: 0 \le y \le 2x - x^2$



$$V = \pi \int_0^2 f^2(x)\, dx = \pi \int_0^2 (2x - x^2)^2\, dx = \pi \int_0^2 4x^2 - 4x^3 + x^4\, dx =$$

$$= \pi \left[4\frac{x^3}{3} - 4\frac{x^4}{4} + \frac{x^5}{5} \right]_0^2 = \pi \left(\frac{32}{3} - 16 + \frac{32}{5} \right)$$

$$\int_0^{\frac{\pi}{4}} \frac{2\cdot\cos x}{(\sin x-2)(\sin^2 x-2\sin x+1)}dx = \int_0^{\frac{\sqrt{2}}{2}} \frac{2}{(y-2)(y^2-2y+1)}dy = 2\int_0^{\frac{\sqrt{2}}{2}} \frac{1}{(y-2)(y-1)^2}dy \overset{*}{=}$$

Subst $y=\sin x$ $\quad x=0 \quad \sin 0=0$

$dy=\cos x\,dx \quad x=\frac{\pi}{4} \quad \sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$$\frac{1}{(y-2)(y-1)^2} = \frac{A}{y-2}+\frac{B}{y-1}+\frac{C}{(y-1)^2} = \frac{1}{y-2}-\frac{1}{y-1}-\frac{1}{(y-1)^2}$$

$$1 = A(y-1)^2+B(y-2)(y-1)+C(y-2)$$

$y=1 \quad 1=C\cdot(-1) \quad C=-1$

$y=2 \quad 1=A \quad A=1$

$y=0 \quad \cancel{1}=A+2B-2C = \cancel{1}+2B+2 \quad B=-1$

$$\overset{*}{=} 2\int_0^{\frac{\sqrt{2}}{2}} \frac{1}{y-2}-\frac{1}{y-1}-\frac{1}{(y-1)^2}dy = 2\left(\left[\ln|y-2|\right]_0^{\frac{\sqrt{2}}{2}} - \left[\ln|y-1|\right]_0^{\frac{\sqrt{2}}{2}} + \left[\frac{1}{y-1}\right]_0^{\frac{\sqrt{2}}{2}}\right) =$$

$$= 2\left(\ln\left(2-\frac{\sqrt{2}}{2}\right)-\ln 2-\ln\left(1-\frac{\sqrt{2}}{2}\right)+0+\frac{1}{\frac{\sqrt{2}}{2}-1}-\frac{1}{-1}\right)$$