Určitý integrál.

1. $$\int_e^3 \frac{1}{x\cdot \ln x}\,dx = \int_1^{\ln 3} \frac{1}{y}\,dy = \left[\ln|y|\right]_1^{\ln 3} = \ln(\ln 3) - \ln 1 = \underline{\underline{\ln(\ln 3)}}$$

Sub. $y = \ln x$ $\quad e \to \ln e = 1$

$dy = \frac{1}{x}\,dx$ $\quad 3 \to \ln 3$

2. $$\int_{-1}^{1} \frac{x}{\sqrt{5-4x}}\,dx = -\frac{1}{4}\int_9^1 \frac{\frac{5-y}{4}}{\sqrt{y}}\,dy = \frac{1}{4}\int_1^9 \frac{1}{4}\left(\frac{5}{\sqrt{y}} - \sqrt{y}\right)dy = \frac{1}{16}\left(5\left[2y^{\frac{1}{2}}\right]_1^9 - \left[\frac{2}{3}y^{\frac{3}{2}}\right]_1^9\right) =$$

$y = 5-4x$ $\quad x = \frac{5-y}{4}$ $\quad -1 \to 9$

$dy = -4dx$ $\quad 1 \to 1$

$$= \frac{10}{16}(3-1) - \frac{2}{3\cdot 16}(27-1) = \frac{5}{4} - \frac{13}{3\cdot 4} = \frac{2}{12} = \underline{\underline{\frac{1}{6}}}$$

3. $$\int_0^1 \frac{x+2}{x^2+2x+5}\,dx = \frac{1}{2}\int_0^1 \frac{2x+2}{x^2+2x+5} + \frac{2}{x^2+2x+5}\,dx = \frac{1}{2}\left[\ln|x^2+2x+5|\right]_0^1 + \int_0^1 \frac{1}{\underbrace{(x^2+2x+1)}_{(x+1)^2} + \underbrace{4}_{2^2}}\,dx$$

$D = 4 - 4\cdot 5 < 0$

$$= \frac{1}{2}(\ln 8 - \ln 5) + \frac{1}{2}\left[\operatorname{arctg}\frac{x+1}{2}\right]_0^1 = \ln\sqrt{\frac{8}{5}} + \frac{1}{2}\left(\operatorname{arctg} 1 - \operatorname{arctg}\frac{1}{2}\right)$$

4. $$\int_0^1 \frac{x+2}{x^2+3x+2}\,dx = \int_0^1 \frac{\cancel{x+2}}{(x+1)\cancel{(x+2)}}\,dx = \left[\ln(x+1)\right]_0^1 = \ln 2 - \ln 1 = \underline{\underline{\ln 2}}$$

$D = 9 - 8 = 1 > 0$ $\quad x^2+3x+2 = (x+1)(x+2)$ $\quad \frac{A}{x+1} + \frac{B}{x+2}$

5. $\displaystyle\int_1^{64} \frac{\sqrt{x}-1}{(1+\sqrt[3]{x})\cdot x}\,dx = \int_1^{2} \frac{t^3-1}{(1+t^2)\cdot t^{\cancel{6}\,1}}\, 6\cancel{t^5}\,dt = 6\int_1^2 \frac{t^3-1}{t^3+t}\,dt = 6\int_1^2 1-\frac{t+1}{(t^2+1)\cdot t}\,dt = *$

Sub. $x = t^6$ $\quad t=\sqrt[6]{x}$ $\quad 1 \to 1$

$dx = 6t^5\,dt$ $\quad 64 \to \sqrt[6]{64} = 2$

$$\begin{aligned} &t^3-1 : (t^3+t) = 1 \\ &\underline{-t^3-t} \\ &-t-1 \end{aligned}$$

$$\frac{t+1}{(t^2+1)\cdot t} = \frac{At+B}{t^2+1} + \frac{C}{t}$$

$t+1 = (At+B)\cdot t + C(t^2+1)$

$t=0$: $\quad 1 = C$

t^2: $\quad 0 = A + C \Rightarrow A=-1$

t^1: $\quad 1 = B$

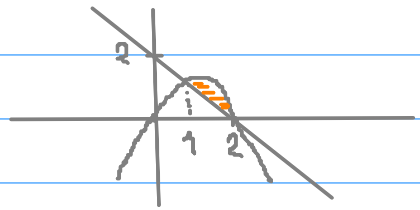
$$* = 6\left([t]_1^2 - \int_1^2 \frac{-t+1}{t^2+1} + \frac{1}{t}\,dt\right) = 6\left(1 - [\ln t]_1^2 - [\operatorname{arctg} t]_1^2 + \frac{1}{2}\left[\ln(t^2+1)\right]_1^2\right) =$$

$$\frac{-t+1}{t^2+1} = \frac{1}{2}\cdot\frac{-2t}{t^2+1} + \frac{1}{t^2+1}$$

$$= 6\left(1 - \ln 2 - \operatorname{arctg} 2 + \operatorname{arctg} 1 + \frac{1}{2}(\ln 5 - \ln 2)\right)$$

6. Obsah oblasti ohraničenej krivkami $\quad y = 2x - x^2$ parabola

$y = -x + 2$ priamka



$2x - x^2 = -x + 2$

$0 = x^2 - 3x + 2$ $\qquad x_1 = 1 \qquad x_2 = 2$

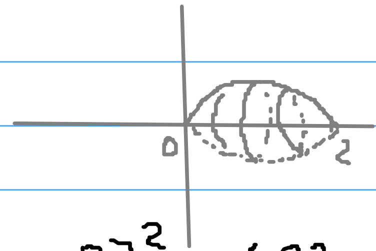
$$\int_1^2 2x-x^2-(-x+2)\,dx = \int_1^2 -x^2+3x-2\,dx = \left[-\frac{x^3}{3}+3\frac{x^2}{2}-2x\right]_1^2 = -\frac{8}{3}+6-4-\left(-\frac{1}{3}+\frac{3}{2}-2\right)=$$

$$=-\frac{7}{3}+4-\frac{3}{2}=\frac{1}{6}$$

7. Objem rotačného telesa

A: $y \leq 2x-x^2$

$y \geq 0$



$$\pi\int_0^2 (2x-x^2)^2\,dx = \pi\int_0^2 4x^2-4x^3+x^4\,dx = \pi\left[4\frac{x^3}{3}-x^4+\frac{x^5}{5}\right]_0^2 = \pi\left(\frac{32}{3}-16+\frac{32}{5}\right)=$$

$$=\pi\,\frac{32(5+3)-16\cdot 15}{15}=\pi\,\frac{16(16-15)}{15}=\frac{16}{15}\pi$$

Skúška P1. $f(x)=\frac{\ln x}{x}$ Vypočítajte D_f, intervaly monotónnosti a lok. ext, inf bod a 1. diferenciál v inf. bode

a) $D_f=R^+$

b) $f'(x)=\frac{\frac{1}{x}\cdot x-\ln x\cdot 1}{x^2}=\frac{1-\ln x}{x^2}$

St. body $1-\ln x=0$

$\ln x = 1$

$$\begin{array}{ccccc} & + & & - & \\ 0 & \nearrow & e & \searrow & \rightarrow \end{array}$$

Rastúca na $(0,e\rangle$

klesajúca na $\langle e,\infty)$ $\Rightarrow$

$\boxed{x=e \text{ je bod OLMAX}}$

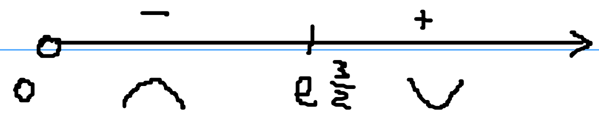
$$f''(x) = \frac{-\frac{1}{x}\cdot x^2 - (1-\ln x)\cdot 2x}{x^4}$$

$f''=0$ $\quad -x - 2x + \ln x \cdot 2x = 0$

$2x \ln x = 3x \qquad x=0 \notin D_f$

$\ln x = \frac{3}{2}$

$f''(1) = -1-2=-3$



$x_0 = e^{\frac{3}{2}}$ - inf. bod

$$df(x, e^{\frac{3}{2}}) = f'(x_0)(x-x_0) = \frac{1-\ln x_0}{x_0^2}(x-x_0) = \frac{1-\frac{3}{2}}{e^3}(x-e^{\frac{3}{2}})$$

Pr 2. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$

$$\frac{2}{4n^2-1} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

a^2-b^2

$2 = A(2n+1) + B(2n-1)$

$n=\frac{1}{2} \qquad 2 = A\cdot 2 \qquad A=1$

$n-\frac{1}{2} \qquad 2 = B(-2) \qquad B=-1$

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} - \frac{1}{2n+1}$$

$$S_n = \frac{1}{1} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \cancel{\ldots} + \cancel{\frac{1}{2n-3}} - \cancel{\frac{1}{2n-1}} + \cancel{\frac{1}{2n-1}} - \frac{1}{2n+1} = 1 - \frac{1}{2n+1}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} 1 - \frac{1}{2n+1} = 1$$

$$\sum_{n=3}^{\infty} \frac{2}{4n^2-1}$$

$$S_n = \frac{1}{5} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{9}} + \cancel{\ldots} + \cancel{\frac{1}{2n-1}} - \frac{1}{2n+1} = \frac{1}{5} - \frac{1}{2n+1}$$

$$S = \lim_{n\to\infty} \frac{1}{5} - \frac{1}{2n+1} = \frac{1}{5}$$

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$$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$$

$$\lim_{n\to\infty} \frac{\frac{3^{2(n+1)}}{(n+1)!}}{\frac{3^{2n}}{n!}} = \lim_{n\to\infty} \frac{\cancel{n!}\ \cancel{3^{2n}} \cdot 9}{\cancel{n!}(n+1)\ \cancel{3^{2n}}} = \lim_{n\to\infty} \frac{9}{n+1} = 0 < 1 \Rightarrow$$

konverguje

Pr 3 $\quad f(x) = \sqrt{x} \cdot \ln^2 x \qquad F(x) = ?$

$$F(x) = \int f(x)\,dx = \int \sqrt{x} \cdot \ln^2 x\,dx = \frac{2}{3}x^{\frac{3}{2}} \cdot \ln^2 x - \int \frac{2}{3}x^{\frac{3}{2}} \cdot 2 \cdot \ln x \cdot \frac{1}{x}\,dx =$$

Per partes $\quad f' = \sqrt{x} \quad f = \frac{2}{3}x^{\frac{3}{2}}$

$\quad g = \ln^2 x \quad g' = 2 \cdot \ln x \cdot \frac{1}{x}$

$$= \frac{2}{3}x^{\frac{3}{2}} \ln^2 x - \frac{4}{3}\int x^{\frac{1}{2}} \cdot \ln x\,dx = \frac{2}{3}x^{\frac{3}{2}} \cdot \ln^2 x - \frac{4}{3}\left(\frac{2}{3}x^{\frac{3}{2}} \cdot \ln x - \int \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{x}\,dx\right) =$$

P. partes $\quad f' = \sqrt{x} \quad f = \frac{2}{3}x^{\frac{3}{2}}$

$\quad g = \ln x \quad g' = \frac{1}{x}$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln^2 x - \frac{8}{9}x^{\frac{3}{2}}\ln x + \frac{8}{9}\int x^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}}\ln^2 x - \frac{8}{9}x^{\frac{3}{2}}\ln x + \frac{16}{27}x^{\frac{3}{2}} + C$$

Pr 4 $$\int_0^{\frac{\pi}{4}} \frac{2\cos x}{(\sin x - 2)(\sin^2 x - 2\sin x + 1)}\,dx$$

Sub. $y = \sin x$

$dy = \cos x\,dx$