

$$\text{Príklad 1. } \lim_{x \rightarrow 1} \frac{\sin(x-1)}{\ln x} = \lim_{x \rightarrow 1} \frac{\cos(x-1)}{\frac{1}{x}} = \lim_{x \rightarrow 1} x \cos(x-1) = 1 .$$

$$\text{Príklad 6. } \lim_{x \rightarrow 0} \frac{x \sin x - x^2}{2 \cos x - 2 + x^2} = \lim_{x \rightarrow 0} \frac{\sin x + x \cos x - 2x}{-2 \sin x + 2x} = \lim_{x \rightarrow 0} \frac{2 \cos x - x \sin x - 2}{-2 \cos x + 2} = \\ \lim_{x \rightarrow 0} \frac{-3 \sin x - x \cos x}{2 \sin x} = \lim_{x \rightarrow 0} \frac{-4 \cos x + x \sin x}{2 \cos x} = -2 .$$

$$\text{Príklad 12. } \lim_{x \rightarrow 0+} x e^{\frac{1}{x}} = \lim_{x \rightarrow 0+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} = \lim_{x \rightarrow 0+} \frac{e^{\frac{1}{x}} (-\frac{1}{x^2})}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0+} e^{\frac{1}{x}} = e^{\lim_{x \rightarrow 0+} \frac{1}{x}} = +\infty .$$

$$\text{Príklad 18. } \lim_{x \rightarrow 0+} (\sin x)^{\operatorname{tg} x} = \lim_{x \rightarrow 0+} e^{\ln(\sin x)^{\operatorname{tg} x}} .$$

Počítajme zvlášť

$$\begin{aligned} \lim_{x \rightarrow 0+} \ln(\sin x) \operatorname{tg} x &= \lim_{x \rightarrow 0+} \frac{\ln(\sin x)}{\cot x} = \\ &= \lim_{x \rightarrow 0+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\frac{1}{\sin^2 x}} = \lim_{x \rightarrow 0+} -\sin x \cos x = 0 \end{aligned}$$

Dosadením do pôvodnej limity dostaneme

$$\lim_{x \rightarrow 0+} (\sin x)^{\operatorname{tg} x} = \lim_{x \rightarrow 0+} e^{\ln(\sin x)^{\operatorname{tg} x}} = e^0 = 1.$$