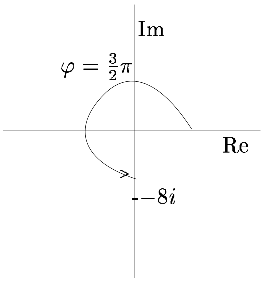
1a. [2 body] Znázornite komplexné číslo $-8i$ a napíšte jeho goniometrický tvar.

1b. [4] Riešte rovnicu $x^3 = -8i$. Výsledok znázornite a vyjadrite v algebraickom tvare.



Najprv pravú stranu znázorníme a z obrázku určíme absolútnu hodnotu $|-8i| = 8$ a argument $\varphi = \frac{3}{2}\pi$.

a. $\underline{-8i = 8(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}i)}$

b. $x^3 = 8\big[\cos\big(\frac{3}{2}\pi + 2k\pi\big) + i\sin\big(\frac{3}{2}\pi + 2k\pi\big)\big]$

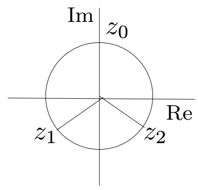
Odmocnením absolútnej hodnoty a delením argumentu dostaneme riešenie:

$$x_k = 2\big[\cos\frac{1}{3}(\frac{3}{2}\pi + 2k\pi\big) + i\sin\frac{1}{3}(\frac{3}{2}\pi + 2k\pi\big)\big]\,, k = 0, 1, 2$$

$$x_0 = 2\big[\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi\big] = 2i$$

$$x_1 = 2\big[\cos(\frac{1}{2}\pi + \frac{2\pi}{3}) + i\sin(\frac{1}{2}\pi + \frac{2\pi}{3})\big] = 2\big[\cos\frac{7}{6}\pi + i\sin\frac{7}{6}\pi\big] = -\sqrt{3} - i$$

$$x_2 = 2\big[\cos(\frac{1}{2}\pi + \frac{4\pi}{3}) + i\sin(\frac{1}{2}\pi + \frac{4\pi}{3})\big] = 2\big[\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi\big] = \sqrt{3} - i$$



2. [6] Funkciu $r(x) = \dfrac{2x^2 + 2x + 2}{x^3 + x^2 + x + 1}$ napíšte ako súčet elementárnych zlomkov nad R.

Menovateľ: $x^3 + x^2 + x + 1 = x^2(x+1) + (x+1) = (x+1)(x^2+1)$.

$$r(x) = \frac{2x^2 + 2x + 2}{(x+1)(x^2+1)} = \frac{a}{x+1} + \frac{bx + c}{x^2+1} \implies 2x^2 + 2x + 2 = a(x^2+1) + (bx+c)(x+1) = (a+b)x^2 + (b+c)x + (a+c)$$

$a + b = b + c = a + c = 2 \implies a = b = c = 1$, $\underline{r(x) = \dfrac{1}{x+1} + \dfrac{x+1}{x^2+1}}$.

3. [6] Riešte sústavu

$$\begin{aligned} 2x_1 - x_2 + x_3 - x_4 &= 0 \\ -x_1 + x_2 + 2x_3 + x_4 &= 3 \\ 3x_1 - x_2 + 4x_3 - x_4 &= 3 \end{aligned}$$

3.

$$\left(\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 0 \\ -1 & 1 & 2 & 1 & 3 \\ 3 & -1 & 4 & -1 & 3 \end{array}\right)_{A_{1*} \leftrightarrow A_{2*}} \sim \left(\begin{array}{cccc|c} -1 & 1 & 2 & 1 & 3 \\ 2 & -1 & 1 & -1 & 0 \\ 3 & -1 & 4 & -1 & 3 \end{array}\right)_{A_{2*} + 2A_{1*}, A_{3*} + 3A_{1*}, A_{1*} \cdot (-1)}$$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & -2 & -1 & -3 \\ 0 & 1 & 5 & 1 & 6 \\ 0 & 2 & 10 & 2 & 12 \end{array}\right)_{A_{3*} - 2A_{2*}} \sim \left(\begin{array}{cccc|c} 1 & -1 & -2 & -1 & -3 \\ 0 & 1 & 5 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)_{A_{1*} + A_{2*}} \sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 5 & 1 & 6 \end{array}\right)$$

$\mathcal{R} = \underline{\{(3 - 3a, 6 - 5a - b, a, b) \colon a, b \in R\}}$.

4. [2] $f(x) = x^4 + 2x^3 + x^2 + x - 5$, napíšte zvyšok po delení $f(x) : (x+1)$

Zvyšok je hodnota polynómu v bode c: $f(-1) = (-1)^4 + 2\cdot(-1)^3 + (-1)^2 + (-1) - 5 = \underline{-6}$