

TÝŽDEŇ 5

1. Vypočítajte limity

a) $\lim_{x \rightarrow a} \frac{x^2 + 3x}{x^2 + x - 6} \quad a = 1, -3 \quad [-1, 3]$

b) $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^3 - 8} \quad [(1/4)]$

c) $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 - 3x - 4} \quad a = 2, -1 \quad [-(3/2), -(3/5)]$

d) $\lim_{x \rightarrow -4} \frac{x^2 + 2x - 8}{x^2 + 4x} \quad [(3/2)]$

e) $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} \quad [3]$

f) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{x} \quad [2]$

g) $\lim_{x \rightarrow -\infty} \frac{x^3 + 1}{x^2 - 3x - 4} \quad [-\infty]$

h) $\lim_{x \rightarrow -\infty} \frac{x^2 + 2x - 8}{2x^2 + 4x - 1} \quad [(1/2)]$

2. Nájdite definičný obor funkcie a vypočítajte limity funkcie v krajných bodoch definičného oboru

a. $f(x) = \sqrt{3x - x^2} \quad \left[\langle 0, 3 \rangle, \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 3^-} f(x) = 0 \right]$

b. $f(x) = \ln \sqrt{3x - x^2} \quad \left[(0, 3) \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 3^-} f(x) = -\infty \right]$

c. $f(x) = \frac{1}{e^{2x} - e^x} \quad \left[\mathbb{R} \setminus \{0\}, \lim_{x \rightarrow 0^-} f(x) = -\infty, \lim_{x \rightarrow 0^+} f(x) = \infty, \lim_{x \rightarrow \infty} f(x) = 0, \lim_{x \rightarrow -\infty} f(x) = -\infty \right]$

d. $f(x) = \frac{2}{3x^2 + x - 2} \quad [\mathbb{R} \setminus \{-1, (2/3)\},]$

$\left[\lim_{x \rightarrow \pm\infty} f(x) = 0, \lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow (2/3)^+} f(x) = \infty, \lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow (2/3)^-} f(x) = -\infty \right]$

e. $f(x) = \arcsin(2 - x) \quad \left[D(f) = \langle 1, 3 \rangle, \lim_{x \rightarrow -1^+} f(x) = \frac{\pi}{2}, \lim_{x \rightarrow 3^-} f(x) = -\frac{\pi}{2} \right]$

f. $f(x) = \arccos \frac{1}{x+1} \quad \left[(-\infty, -2) \cup \langle 0, \infty \rangle, \lim_{x \rightarrow \pm\infty} f(x) = \frac{\pi}{2}, \lim_{x \rightarrow -2^-} f(x) = \pi, \lim_{x \rightarrow 0^+} f(x) = 0 \right]$

3. Vypočítajte limity.

a) $\lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{7 - x} - 2} \quad [-4]$

b) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}} \quad [-1]$

c) $\lim_{x \rightarrow -1} \frac{\sqrt{2x + 6} - 2}{x^2 - x - 2} \quad [-(1/6)]$

d) $\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 1} - \sqrt{2}}{x^2 + x} \quad [(\sqrt{2}/2)]$

4. Vypočítajte limity.

a) $\lim_{x \rightarrow \infty} e^x \arctg x \quad [\infty]$

b) $\lim_{x \rightarrow 0^+} \frac{\ln x}{x} \quad [-\infty]$

c) $\lim_{x \rightarrow \infty} \frac{\sin x}{e^x} \quad [0]$

d) $\lim_{x \rightarrow 0^+} \frac{\cos x}{\ln x} \quad [0]$

e) $\lim_{x \rightarrow \infty} \frac{\sin x - 1}{x} \quad [0]$

f) $\lim_{x \rightarrow \infty} \cos x \cdot \arctg \frac{1}{x} \quad [0]$

g) $\lim_{x \rightarrow \infty} \sin x \cdot \operatorname{arccotg} x \quad [0]$

h) $\lim_{x \rightarrow 0} x \operatorname{arccotg} \frac{1}{x} \quad [0]$

i) $\lim_{x \rightarrow \infty} \frac{x^3 + x}{3 - 2x^3} \quad [-\frac{1}{2}]$

j) $\lim_{x \rightarrow \infty} \frac{\sqrt{x} + 1}{x + \sqrt{x} - 1} \quad [0]$