1. Napíšte množinu všetkých riešení sústavy, ktorej rozšírená matica je

a) [5 bodov] v $\mathbb{R}$: $\left(\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 1 & 3 & -1 \end{array}\right) \sim_{r_2-r_3} \left(\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 & 3 & -1 \end{array}\right)$

parametre: $x_2 = a$, $x_5 = b$, $x_6 = c$,

$P = \{(-1 + a - b + c\,,\ a\,,\ 2 + b\,,\ -1 - b - 3c\,,\ b\,,\ c) \colon a, b, c \in \mathbb{R}\}$

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b) [4] v $\mathbb{C}$: $\left(\begin{array}{cc|c} 1+i & 1 & i \\ 0 & i & i \end{array}\right)$

$ix_2 = i \implies x_2 = 1$, $(1+i)x_1 + 1 = i \implies x_1 = \frac{i-1}{i+1}$, $x_1 = i$, $P = \{(i, 1)\}$

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2. [5] Riešte sústavu (v R)

$$\begin{aligned} 3x - 7y - 3z &= 2 \\ -2x + 4y + z &= -1 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 3 & -7 & -3 & 2 \\ -2 & 4 & 1 & -1 \end{array}\right) \sim_{r_1+r_2} \left(\begin{array}{ccc|c} 1 & -3 & -2 & 1 \\ -2 & 4 & 1 & -1 \end{array}\right) \sim_{r_2+2r_1} \left(\begin{array}{ccc|c} 1 & -3 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right)$$

$z = p$, $y = -\frac{1+3p}{2}$, $\quad x = 1 + 3y + 2z = 1 - \frac{3}{2} - \frac{9}{2}p + 2p$,

$P = \{(-\frac{1}{2} - \frac{5}{2}p\,,\ -\frac{1+3p}{2}\,,\ p) \colon p \in \mathbb{R}\}$

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3. [7] $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Vypočítajte AB, BA, $\det(AB)$, $(AB)^{-1}$, $(BA)^{-1}$,

$AB = \begin{pmatrix} 4 & 2 & 1 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$, $BA = \begin{pmatrix} 3 & 5 \\ 1 & 3 \end{pmatrix}$, $\det(AB) = 0 \implies \nexists (AB)^{-1}$,

$\det(BA) = 4$, $(BA)^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -5 \\ -1 & 3 \end{pmatrix}$

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4. [4] Zistite, či sa vektor $\mathbf{b} = (1, 2, 3)$ dá vyjadriť ako lineárna kombinácia vektorov $(1, 1, 1)$, $(1, 1, -1)$, $(0, 0, 2)$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & -1 & 2 & 3 \end{array}\right) \sim_{r_2-r_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 2 & 3 \end{array}\right) \implies \text{nedá sa}$$

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5. [5] Nájdite $a \in \mathbb{R}$, pre ktoré je $c = 2$ koreňom polynómu $f(x) = x^5 - ax^4 - 2x^3 + 5ax^2 + 5ax - 2a^2$.

	1	$-a$	-2	$5a$	$5a$	$-2a^2$
2		2	$4-2a$	$4-4a$	$8+2a$	$16+14a$
	1	$2-a$	$2-2a$	$4+a$	$8+7a$	$-2a^2+14a+16$

$a^2 - 7a - 8 = 0$, $D = 49 + 32 = 81$, $a_{1,2} = \frac{7 \pm 9}{2}$, $a_1 = 8$, $a_2 = -1$