

Pr1A. [14 bodov] $f(x) = 2x^5 + x^3 + 2x^2 + x + 2$, $g(x) = x^4 + x^3 + x + 2 \in P(\mathbb{Z}_3)$.
 Určte $a(x), b(x) \in P(\mathbb{Z}_3)$, pre ktoré $\gcd(f, g) = a(x)f(x) + b(x)g(x)$

$$\begin{aligned}
 & (2x^5 + x^3 + 2x^2 + x + 2) : (x^4 + x^3 + x + 2) = 2x + 1 \\
 & \underline{-(2x^5 + 2x^4 + 2x^2 + x)} \\
 & \quad x^4 + x^3 + 2 \\
 & \quad \underline{-(x^4 + x^3 + x + 2)} \\
 & \quad \quad 2x = f_2 = f - (2x + 1)g \\
 & \quad (x^4 + x^3 + x + 2) : (2x) = 2x^3 + 2x^2 + 2 \\
 & \quad \underline{-x^4} \\
 & \quad \quad x^3 + x + 2 \\
 & \quad \quad \underline{-x^3} \\
 & \quad \quad \quad x + 2 \\
 & \quad \quad \quad \underline{-x} \\
 & \quad \quad \quad \quad 2 = \gcd(f, g) = g(x) - (2x^3 + 2x^2 + 2)f_2 = g(x) - (2x^3 + 2x^2 + 2)[f(x) - (2x + 1)g(x)] \\
 & \gcd(f, g) = \underbrace{(x^3 + x^2 + 1)}_{a(x)} f(x) + \underbrace{(x^4 + 2x^2 + x)}_{b(x)} g(x)
 \end{aligned}$$

Pr1B. [14] $f(x) = 2x^5 + x^4 + 2x^2 + x + 2$, $g(x) = x^4 + x^3 + x + 2 \in P(\mathbb{Z}_3)$.
 Určte $a(x), b(x) \in P(\mathbb{Z}_3)$, pre ktoré $\gcd(f, g) = a(x)f(x) + b(x)g(x)$

$$\begin{aligned}
 & (2x^5 + x^4 + 2x^2 + x + 2) : (x^4 + x^3 + x + 2) = 2x + 2 \\
 & \underline{-(2x^5 + 2x^4 + 2x^2 + x)} \\
 & \quad 2x^4 + 2 \\
 & \quad \underline{-(2x^4 + 2x^3 + 2x + 1)} \\
 & \quad \quad x^3 + x + 1 = f_2 = f - (2x + 2)g = f + (x + 1)g \\
 & \quad (x^4 + x^3 + x + 2) : (x^3 + x + 1) = x + 1 \\
 & \quad \underline{-(x^4 + x^2 + x)} \\
 & \quad \quad x^3 + 2x^2 + 2 \\
 & \quad \quad \underline{-(x^3 + x + 1)} \\
 & \quad \quad \quad 2x^2 + 2x + 1 = f_3 = g - (x + 1)f_2 \\
 & \quad (x^3 + x + 1) : (2x^2 + 2x + 1) = 2x + 1 \\
 & \quad \quad \underline{-(x^3 + x^2 + 2x)} \\
 & \quad \quad \quad 2x^2 + 2x + 1, \text{zv. } 0 \implies \gcd(f, g) = f_3 \\
 & = g - (x + 1)f_2 = g - (x + 1)[f + (x + 1)g] = (2x + 2)f + [1 - (x + 1)^2]g \\
 & = \underbrace{(2x + 2)}_{a(x)} f(x) + \underbrace{(2x^2 + x)}_{b(x)} g(x)
 \end{aligned}$$

Pr.2A [12 bodov] $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$. Vypočítajte

a) $\text{tr}A$, b) vlastné čísla matice A , c) maticu P a diagonálnu maticu D , pre ktorú $A = PDP^T$.

a) $\text{tr}A = 5$, b) Očividne $\lambda_1 = 1$ aj $\lambda_2 = 0$ sú vlastné čísla matice A , teda $\lambda_3 = 4$, prípadne počítame

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 2 \\ 0 & 2 & 2-\lambda \end{vmatrix} = -\lambda(1-\lambda)(4-\lambda) \text{ a dostaneme } \sigma(A) = (1, 0, 4).$$

c) vlastné vektory: pre $\lambda_1 = 1$ je $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, pre $\lambda_2 = 0$ $A - \lambda_2 I = A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$,

$\lambda_3 = 4$ $A - \lambda_3 I = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\|\mathbf{v}_2\| = \|\mathbf{v}_3\| = \sqrt{2}$, $\|\mathbf{v}_1\| = 1$.

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Pr.2B [12 bodov] $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$. Vypočítajte

a) $\text{tr}A$, b) vlastné čísla matice A , c) maticu P a diagonálnu maticu D , pre ktorú $A = PDP^T$.

a) $\text{tr}A = 3$, b) Očividne $\lambda_1 = -1$ aj $\lambda_2 = 0$ sú vlastné čísla matice A , teda $\lambda_3 = 4$, prípadne počítame

$$\begin{vmatrix} -1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 2 \\ 0 & 2 & 2-\lambda \end{vmatrix} = \lambda(-1-\lambda)(\lambda-4) \text{ a dostaneme } \sigma(A) = (-1, 0, 4).$$

c) vlastné vektory: pre $\lambda_1 = -1$ je $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, pre $\lambda_2 = 0$ $A - \lambda_2 I = A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$,

$\lambda_3 = 4$ $A - \lambda_3 I = \begin{pmatrix} -5 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\|\mathbf{v}_2\| = \|\mathbf{v}_3\| = \sqrt{2}$, $\|\mathbf{v}_1\| = 1$.

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Pr3A) [14]

Napíšte maticu lineárneho operátora $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ vzhľadom na štandardné bázy, ak T zobrazí bod A na bod s ním súmerne združený podľa roviny $\rho \equiv 2x + 2y + z = 0$.

Stĺpce matice $T_{\mathcal{E}}$ tvoria obrazy štandardnej bázy.

$$\begin{aligned} \mathbf{n} &= (2, 2, 1) \perp \rho, T(1, 0, 0) = (x, y, z) \implies (x - 1, y, z) = t(2, 2, 1) \implies T(1, 0, 0) = (1 + 2t, 2t, t) \\ \mathbf{e}_1 = A &= (1, 0, 0), B = (x, y, z) \text{ dist}(A, \rho) = \text{dist}(B, \rho) \implies \\ 2 &= |2(1 + 2t) + 2 \cdot 2t + t| = |2 + 9t| \implies 2 + 9t = \pm 2, \\ 2 + 9t &= 2 \implies t = 0 \text{ zodpovedá bodu } A, \\ 2 + 9t &= -2 \implies t = -\frac{4}{9} \implies 1 + 2t = \frac{9-8}{9} = \frac{1}{9}, 2t = -\frac{8}{9}, \text{ t.j. } B = \frac{1}{9}(1, -8, -4) \\ T_{\mathcal{E}} = P &\implies P_{*1} = \frac{1}{11} \begin{pmatrix} 1 \\ -8 \\ -4 \end{pmatrix}. \end{aligned}$$

Podobný postup zopakujeme pre body $T\mathbf{e}_2 = T(0, 1, 0)$ a $T\mathbf{e}_3 = T(0, 0, 1)$ a dostaneme

$$T(0, 1, 0) = (x, y, z) \implies (x, y - 1, z) = t(2, 2, 1) \implies 9t + 2 = -2 \implies t = -\frac{4}{9},$$

$$x = 2t = -\frac{8}{9}, y = \frac{1}{9}, z = t = -\frac{4}{9}.$$

$$T(0, 0, 1) = (x, y, z) \implies x = -\frac{4}{9}, y = -\frac{4}{9}, z = \frac{7}{9}.$$

$$T_{\mathcal{E}} = \frac{1}{9} \begin{pmatrix} 1 & -8 & -4 \\ -8 & 1 & -4 \\ -4 & -4 & 7 \end{pmatrix}$$

Pr3B)

Napíšte maticu lineárneho operátora $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ vzhľadom na štandardné bázy, ak T zobrazí bod A na bod s ním súmerne združený podľa roviny $\rho \equiv 2x + 2y - z = 0$.

Maticu môžeme vypočítať podobne ako v Pr3A. Ukážeme si ešte iný postup.

Pre tento operátor je ľahké nájsť bázu $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ zloženú z vlastných vektrov:

$$\mathbf{b}_1 = \mathbf{n} = (2, 2, -1) \perp \rho \implies T\mathbf{b}_1 = -\mathbf{b}_1, \text{ príslušné vlastné číslo je } -1.$$

Každý vektor $\mathbf{u} \parallel \rho$, resp. bod $A \in \rho$ sa zobrazí na seba (je vlastný s vlastným číslom =1). Zvolíme napr.

$$\mathbf{b}_2 = (1, -1, 0) \text{ a } \mathbf{b}_3 = (0, 1, 2).$$

$$T_{\mathcal{B}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \text{ ak } P = \begin{pmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ -1 & 0 & 2 \end{pmatrix}, \text{ tak } T_{\mathcal{E}} = PT_{\mathcal{B}}P^{-1}.$$

$$\text{Rozvojom podľa } R_3: \begin{vmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = -1 - 8 = -9, P^{-1} = -\frac{1}{9} \begin{pmatrix} -2 & -2 & 1 \\ -5 & 4 & -2 \\ -1 & -1 & -4 \end{pmatrix}$$

$$T_{\mathcal{E}} = -\frac{1}{9} \begin{pmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & -2 & 1 \\ -5 & 4 & -2 \\ -1 & -1 & -4 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 2 & -1 & 0 \\ 2 & 1 & -1 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} -2 & -2 & 1 \\ -5 & 4 & -2 \\ -1 & -1 & -4 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 1 & -8 & 4 \\ -8 & 1 & 4 \\ 4 & 4 & 7 \end{pmatrix}$$

$$T_{\mathcal{E}} = \frac{1}{9} \begin{pmatrix} 1 & -8 & 4 \\ -8 & 1 & 4 \\ 4 & 4 & 7 \end{pmatrix}$$

Pr4A [14] $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3, 2x_1 + 2x_2 - x_3, 4x_1 + x_2)$, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, $\mathbf{b}_1 = (0, 1, 1)$, $\mathbf{b}_2 = (1, 0, 0)$, $\mathbf{b}_3 = (0, 2, 1)$. \mathcal{E} je štandardná báza. Určte

a) matice $T_{\mathcal{E}\mathcal{E}}$, $T_{\mathcal{E}\mathcal{B}}$, $T_{\mathcal{B}\mathcal{B}}$

b) bázu a dimenziu jadra a dimenziu oboru hodnôt operátora T .

$$\begin{aligned} \text{a) } T\mathbf{b}_1 &= T(0, 1, 1) = (0, 1, 1), \quad T\mathbf{b}_2 = T(1, 0, 0) = (2, 2, 4), \quad T\mathbf{b}_3 = T(0, 2, 1) = (-1, 3, 2), \\ T\mathbf{e}_1 &= T(1, 0, 0) = (2, 2, 4), \quad T\mathbf{e}_2 = T(0, 1, 0) = (-1, 2, 1), \quad T\mathbf{e}_3 = T(0, 0, 1) = (1, -1, 0). \end{aligned}$$

$$\left(\begin{array}{ccc|ccc} \mathcal{B} & & & T\mathcal{E} & & T\mathcal{B} \\ 0 & 1 & 0 & 2 & -1 & 1 & 0 & 2 & -1 \\ 1 & 0 & 2 & 2 & 2 & -1 & 1 & 2 & 3 \\ 1 & 0 & 1 & 4 & 1 & 0 & 1 & 4 & 2 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 4 & 1 & 0 & 1 & 4 & 2 \\ 0 & 1 & 0 & 2 & -1 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & -2 & 1 & -1 & 0 & -2 & 1 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 0 & 1 & 1 & 6 & 1 \\ 0 & 1 & 0 & 2 & -1 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & -2 & 1 & -1 & 0 & -2 & 1 \end{array} \right)$$

$$T_{\mathcal{E}\mathcal{E}} = \begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 4 & 1 & 0 \end{pmatrix}, \quad T_{\mathcal{E}\mathcal{B}} = \begin{pmatrix} 6 & 0 & 1 \\ 2 & -1 & 1 \\ -2 & 1 & -1 \end{pmatrix}, \quad T_{\mathcal{B}\mathcal{B}} = \begin{pmatrix} 1 & 6 & 1 \\ 0 & 2 & -1 \\ 0 & -2 & 1 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 4 & 1 & 0 \end{pmatrix} \sim_{r_3-r_2} \begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{pmatrix}, \quad x_3 = 3t, \quad x_2 = 2t, \quad x_1 = -\frac{1}{2}t$$

$\text{Ker } T = \{(-\frac{1}{2}t, 2t, 3t); t \in \mathbb{R}\}$, Báza: $\{(-\frac{1}{2}, 2, 3)\}$, $\dim \text{Ker } T = 1$, $\dim \text{Ran } T = 3 - 1 = 2$.

Pr4B [14] $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3, 2x_1 + 2x_2 - x_3, 4x_1 + x_2)$, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, $\mathbf{b}_1 = (0, 1, 1)$, $\mathbf{b}_2 = (0, 2, 1)$, $\mathbf{b}_3 = (1, 0, 0)$. \mathcal{E} je štandardná báza. Určte

a) matice $T_{\mathcal{E}\mathcal{E}}$, $T_{\mathcal{E}\mathcal{B}}$, $T_{\mathcal{B}\mathcal{B}}$

b) bázu a dimenziu jadra a dimenziu oboru hodnôt operátora T .

$$\begin{aligned} \text{a) } T\mathbf{b}_1 &= T(0, 1, 1) = (0, 1, 1), \quad T\mathbf{b}_2 = T(0, 2, 1) = (-1, 3, 2), \quad T\mathbf{b}_3 = T(1, 0, 0) = (2, 2, 4), \\ T\mathbf{e}_1 &= T(1, 0, 0) = (2, 2, 4), \quad T\mathbf{e}_2 = T(0, 1, 0) = (-1, 2, 1), \quad T\mathbf{e}_3 = T(0, 0, 1) = (1, -1, 0). \end{aligned}$$

$$\left(\begin{array}{ccc|ccc} \mathcal{B} & & & T\mathcal{E} & & T\mathcal{B} \\ 0 & 0 & 1 & 2 & -1 & 1 & 0 & -1 & 2 \\ 1 & 2 & 0 & 2 & 2 & -1 & 1 & 3 & 2 \\ 1 & 1 & 0 & 4 & 1 & 0 & 1 & 2 & 4 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 4 & 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & 0 & -2 & 1 & -1 & 0 & 1 & -2 \\ 0 & 0 & 1 & 2 & -1 & 1 & 0 & -1 & 2 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 0 & 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & -2 & 1 & -1 & 0 & 1 & -2 \\ 0 & 0 & 1 & 2 & -1 & 1 & 0 & -1 & 2 \end{array} \right)$$

$$T_{\mathcal{E}\mathcal{E}} = \begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 4 & 1 & 0 \end{pmatrix}, \quad T_{\mathcal{E}\mathcal{B}} = \begin{pmatrix} 6 & 0 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}, \quad T_{\mathcal{B}\mathcal{B}} = \begin{pmatrix} 1 & 1 & 6 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 4 & 1 & 0 \end{pmatrix} \sim_{r_3-r_2} \begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{pmatrix}, \quad x_3 = 3t, \quad x_2 = 2t, \quad x_1 = -\frac{1}{2}t$$

$\text{Ker } T = \{(-\frac{1}{2}t, 2t, 3t); t \in \mathbb{R}\}$, Báza: $\{(-\frac{1}{2}, 2, 3)\}$, $\dim \text{Ker } T = 1$, $\dim \text{Ran } T = 3 - 1 = 2$.

Pr5A [12]

Nájdite rovnicu priamky $y(x) = kx + q$, pre ktorú odchýlka $\delta = \sum_{i=-2}^2 |y(x_i) - y_i|^2$ najmenšia. y_i sú hodnoty namerané v bodoch x_i z tabuľky

A)	<table border="1" style="border: none;"> <tr> <td style="padding: 2px 10px;">x_i</td> <td style="padding: 2px 10px;">-2</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">2</td> </tr> <tr> <td style="padding: 2px 10px;">y_i</td> <td style="padding: 2px 10px;">2</td> <td style="padding: 2px 10px;">1,5</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> </tr> </table>	x_i	-2	-1	0	1	2	y_i	2	1,5	0	0	-1
x_i	-2	-1	0	1	2								
y_i	2	1,5	0	0	-1								

B)	<table border="1" style="border: none;"> <tr> <td style="padding: 2px 10px;">x_i</td> <td style="padding: 2px 10px;">-2</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">2</td> </tr> <tr> <td style="padding: 2px 10px;">y_i</td> <td style="padding: 2px 10px;">-2</td> <td style="padding: 2px 10px;">-1,5</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> </tr> </table>	x_i	-2	-1	0	1	2	y_i	-2	-1,5	0	0	1
x_i	-2	-1	0	1	2								
y_i	-2	-1,5	0	0	1								

Načrtnite graf priamky p spolu s bodmi $[x_i, y_i]$ a vypočítajte odchýlku δ .

Riešenie A) (príklad B) rovnako, len na niekoľkých miestach zmena +/-)

$$\begin{aligned}
 -2k + q &= 2 \\
 -k + q &= 1,5 \\
 q &= 0 \\
 k + q &= 0 \\
 2k + q &= -1
 \end{aligned}
 \quad A = \begin{pmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1,5 \\ 0 \\ 0 \\ -1 \end{pmatrix} \quad A^T A \begin{pmatrix} k \\ q \end{pmatrix} = A^T \mathbf{b}$$

$$\begin{pmatrix} 10 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} k \\ q \end{pmatrix} = \begin{pmatrix} -7,5 \\ 2,5 \end{pmatrix} \implies k = -0,75, \quad q = 0,5$$

$$\text{A) } p \equiv y = -0,75x + 0,5, \quad \text{B) } p \equiv y = 0,75x - 0,5$$

$$y(-2) = 2 \cdot 0,75 + 0,5 = 2 = y_{-2};$$

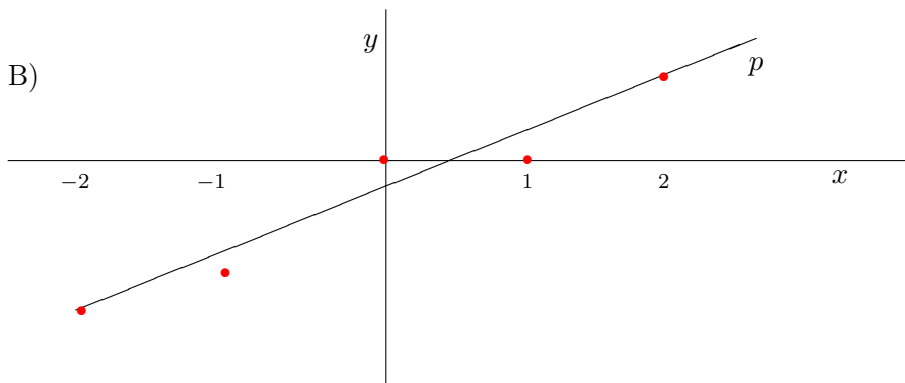
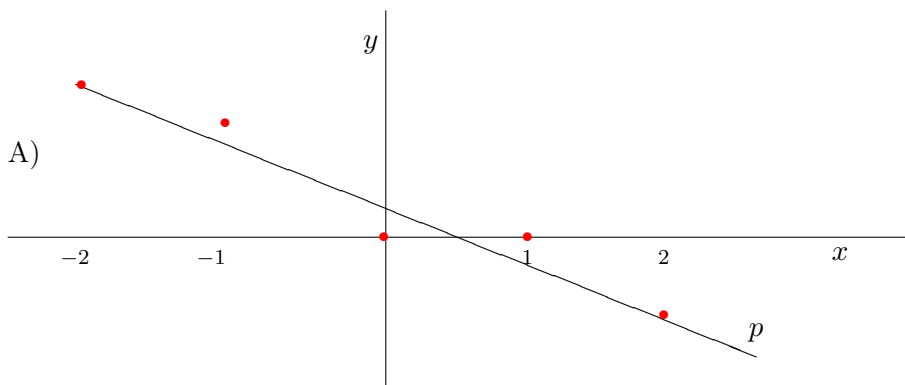
$$y(-1) = 0,75 + 0,5 = 1,25 < y_{-1} = 1,5;$$

$$y(0) = 0,5 > y_0 = 0;$$

$$y(1) = -0,25 < y_1 = 0;$$

$$y(-2) = -1 = y_2$$

$$\delta = 0^2 + 0,25^2 + 0,5^2 + 0,25^2 + 0^2 = 0,25 + 2 \cdot 0,0625 = 0,3750$$



- Pr6A) [14] Napíšte a) všetky ireducibilné polynómy nad Z_2 stupňa 2,3,4
 b) koľko prvkov má pole $F_1 = P(Z_2)/(x^3 + x^2 + 1)$,
 c) koľko prvkov má pole $F_2 = P(Z_2)/(x^2 + x)$
 d) rozklad polynómu $f(x) = x^5 + x^4 + 1 \in P(Z_2)$ na súčin ireducibilných polynómov
 e) pole, ktoré má 18 prvkov

a) $x^2 + x + 1$,
 $x^3 + x + 1$, $x^3 + x^2 + 1$,
 $x^4 + x + 1$, $x^4 + x^3 + 1$, $x^4 + x^3 + x^2 + x + 1$

- b) F_1 má 8 prvkov, c) $x^2 + x = x(x + 1)$ nie je ireducibilný, F_2 nie je pole.
 d) $f(0) = f(1) = 1$, $(x^5 + x^4 + 1) : (x^2 + x + 1) = x^3 + x + 1$, $f(x) = \underline{(x^2 + x + 1)(x^3 + x + 1)}$
 e) $18 = 2 \cdot 3^2 \implies \cancel{A}$

- Pr6B) [14] Napíšte a) všetky ireducibilné polynómy nad Z_2 stupňa 2,3,4
 b) koľko prvkov má pole $F_1 = P(Z_2)/(x^3 + x + 1)$,
 c) koľko prvkov má pole $F_2 = P(Z_2)/(x^2 + x)$
 d) rozklad polynómu $f(x) = x^5 + x^4 + x^3 + x + 1 \in P(Z_2)$ na súčin ireducibilných polynómov
 e) pole, ktoré má 36 prvkov

a) $x^2 + x + 1$,
 $x^3 + x + 1$, $x^3 + x^2 + 1$,
 $x^4 + x + 1$, $x^4 + x^3 + 1$, $x^4 + x^3 + x^2 + x + 1$

- b) F_1 má 8 prvkov, c) $x^2 + x = x(x + 1)$ nie je ireducibilný, F_2 nie je pole.
 d) $f(0) = f(1) = 1$, $(x^5 + x^4 + x^3 + x + 1) : (x^2 + x + 1) = x^3$ zv.: $x + 1$, $f(x)$ je ireducibilný
 e) $36 = 2^2 \cdot 3^2 \implies \cancel{A}$