Fourierove transformácie

$$\eta(t) = \begin{cases} 1 & t \geq 0\,, \\ 0\,, & t < 0\,, \end{cases} \qquad \operatorname{sinc} x = \begin{cases} \frac{\sin x}{x}\,, & x \neq 0 \\ 1\,, & x = 0 \end{cases}\,, \quad f * g(x) = \int_{-\infty}^{\infty} f(x-t) g(t)\,\mathrm{d}\,t$$

$f(t)$	$F(i\omega)$
$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\omega)e^{i\omega t}\,\mathrm{d}\,\omega$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,\mathrm{d}\,t$
$e^{-at}\eta(t)\,,\ a > 0$	$\frac{1}{a+i\omega}$
$\eta(t+T) - \eta(t-T)\,, (T > 0)$	$2T \operatorname{sinc} \omega T$
$e^{-a\|t\|}\,,\ a > 0$	$\frac{2a}{a^2+\omega^2}$
$\frac{\mathrm{d}^n}{\mathrm{d}\,t^n} f(t)$	$(i\omega)^n F(i\omega)$
$(-it)^n f(t)$	$\frac{\mathrm{d}^n}{\mathrm{d}\,\omega^n} F(i\omega)$
$f(t-\tau)\,,\ \tau \in R$	$e^{-i\omega\tau} F(i\omega)$
$e^{i\omega_0 t} f(t)$	$F(i(\omega - \omega_0))$
$F(it)$	$2\pi f(-\omega)$
$f * g(t)$	$F(i\omega)G(i\omega)$
$f(t)g(t)$	$F * G(i\omega)$

PRÍKLADY

Vypočítajte Fourierove transformácie funkcií (a, A, T sú dané kladné čísla).

1. $f(t) = \eta(t)e^{-at}$,
2. $f(t) = e^{-a|t|}$,
3. $f(t) = \begin{cases} A, & (-T \le t \le 0) \\ -A, & (0 < t \le T) \end{cases}$
3. $f(t) = \begin{cases} (A/T)t + A & (-T \le t \le 0) \\ (-A/T)t + A, & (0 < t \le T) \end{cases}$
4. $f(t) = \eta(t)e^{-at}\sin\omega_0 t$ (Pomôcka: $2i\sin\alpha = e^{i\alpha} - e^{-i\alpha}$)
5. $f(t) = \begin{cases} 1 & (1 \le |t| \le 2) \\ 0 & (\text{pre ostatné } t \in R \end{cases}$; $g(t) = tf(t)$
6. $f(t) = \sin\omega_0 t[\eta(t + \frac{1}{2}T) - \eta(t - \frac{1}{2}T)$
7. $f(t) = \begin{cases} 0, & t \in (-\infty, -1\rangle \cup (7, \infty) \\ 2, & t \in (-1, 3\rangle \\ -1, & t \in (3, 5\rangle \\ 1, & t \in (5, 7\rangle \end{cases}$
8. $f(t) = \begin{cases} 0, & |t| > 1 \\ t, & |t| \le 1 \end{cases}$
9. $f(t) = \begin{cases} \frac{\sin t}{t}, & t \ne 0 \\ 1, & t = 0 \end{cases}$, $g(t) = \big(f(t)\big)^2$. Vypočítajte $\int_{-\infty}^{\infty} f(t)\,\mathrm{d}\,t$, $\int_{-\infty}^{\infty} g(t)\,\mathrm{d}\,t$.
10. $f(t) = \frac{1}{1+t^2}$, $g(t) = \frac{t}{(t^2+1)^2}$, $h(t) = \frac{1}{1+(t-2)^2}$
11. Nájdite funkciu $G(i\omega)$, takú, že ak $y(t), u(t)$ spĺňa diferenciálnu rovnicu

(a)
$$\frac{\mathrm{d}^2\,y(t)}{\mathrm{d}\,t^2} + 3\frac{\mathrm{d}\,y(t)}{\mathrm{d}\,t} + 7y(t) = 3\frac{\mathrm{d}\,u(t)}{\mathrm{d}\,t} + 2u(t)\,,$$

(b)
$$\frac{\mathrm{d}^2\,y(t)}{\mathrm{d}\,t^2} + 4\frac{\mathrm{d}\,y(t)}{\mathrm{d}\,t} + 5y(t) = \frac{\mathrm{d}\,u(t)}{\mathrm{d}\,t} - 4u(t)\,,$$

tak $Y(i\omega) = G(i\omega)U(i\omega)$.

12. Pomocou Fourierovej transformácie riešte parciálnu diferenciálnu rovnicu

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}\,, \quad u(x,0) = \frac{1}{1+x^2}\,,\ x \in R\,.$$

13. Pomocou Fourierových radov riešte parciálnu diferenciálnu rovnicu

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial^2 u(x,t)}{\partial x^2}\,, x \in \langle 0, \pi\rangle\,, t \ge 0$$

s okrajovými a začiatočnými podmienkami

$$u(0,t) = u(\pi,t) = 0\,, \qquad t \ge 0$$
$$u(x,0) = f_1(x) = \begin{cases} x\,, & 0 \le x \le (\pi/2) \\ \pi - x\,, & (\pi/2) \le x \le \pi\,. \end{cases}$$
$$\frac{\partial u(x,0)}{\partial t} = 0$$