

1. doméce úloha - RIEŠENIE

① Dôkaz sporom. Nech tvrdenie neplatí, teda $\forall i \in \{1, 2, \dots, n\}$ je $a_i > \frac{M}{i}$. Potom $ia_i > M$, a teda

$$a_1 + 2a_2 + 3a_3 + \dots + na_n > nM.$$

Potom

$$M = \frac{a_1 + 2a_2 + 3a_3 + \dots + na_n}{n} > \frac{nM}{n} = M.$$

Maíme teda $M > M$, čo je spor.

② 1° $n=1$ $1^3 = \left[\frac{1(1+1)}{2}\right]^2$, čo platí

2° $V(n) \Rightarrow V(n+1)$

Nech platí

$$\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Potom

$$\begin{aligned} \sum_{i=1}^{n+1} i^3 &= \sum_{i=1}^n i^3 + (n+1)^3 = \left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3 = \\ &= (n+1)^2 \left[\frac{n^2}{4} + (n+1)\right] = (n+1)^2 \frac{n^2 + 4n + 4}{4} = \\ &= \frac{(n+1)^2 (n+2)^2}{4} = \left[\frac{(n+1)(n+2)}{2}\right]^2 \end{aligned}$$

1° \wedge 2° \Rightarrow tvrdenie platí