$\forall m, n \in \mathbb{Z} : 2 \mid m \wedge 2 \mid n \Rightarrow 2 \mid (m+n)$

$2 \mid m \wedge 2 \mid n \Rightarrow m = 2k \wedge n = 2l \ , \ k, l \in \mathbb{Z} \Rightarrow$

$\Rightarrow m+n = 2k + 2l = 2(k+l) \Rightarrow 2 \mid (m+n)$

$\forall x, y \in \mathbb{R} : x \in \mathbb{Q} \wedge y \in \mathbb{Q} \Rightarrow x+y \in \mathbb{Q}$

$x \in \mathbb{Q} \wedge y \in \mathbb{Q} \Rightarrow x = \frac{p}{q} \wedge y = \frac{r}{s} \ , \ p, q, r, s \in \mathbb{Z}$

$\Rightarrow x+y = \frac{p}{q} + \frac{r}{s} = \frac{ps + qr}{qs} = \frac{m}{n} \ , \ m, n \in \mathbb{Z}$

$\Rightarrow x+y \in \mathbb{Q}$

X, Y, Z

$X \subseteq Y \Rightarrow X \cup Z \subseteq Y \cup Z$

$a \in X \cup Z \Rightarrow a \in Y \cup Z$

Nech $a \in X \cup Z \Rightarrow a \in X \vee a \in Z$

1) $\Rightarrow a \in X \Rightarrow a \in Y \Rightarrow a \in Y \cup Z$

2) $\Rightarrow a \in Z \Rightarrow a \in Y \cup Z$

$\Rightarrow a \in Y \cup Z$

$\forall n \in \mathbb{Z} : 5 \nmid n^3 \Rightarrow 5 \nmid n$

$a \mid b$

$a \nmid b$

$n^3 = n \cdot n^2$

$5 \mid n \Rightarrow 5 \mid n^3$

$5 \mid n \Rightarrow n = 5k, \ k \in \mathbb{Z} \Rightarrow n^3 = (5k)^3 = 125k^3 = 5 \cdot 25k^3 \Rightarrow$

$\Rightarrow n^3 = 5l \ , \ l \in \mathbb{Z} \Rightarrow 5 \mid n^3$

$\forall x, y, z \in \mathbb{R} : x+y+z \geq 3 \Rightarrow (x \geq 1 \vee y \geq 1 \vee z \geq 1)$

$$(x<1 \wedge y<1 \wedge z<1) \Rightarrow x+y+z<3$$

$$\overline{a \vee b \vee c} = \bar{a} \wedge \bar{b} \wedge \bar{c}$$

$$x<1 \wedge y<1 \wedge z<1 \Rightarrow x+y+z<1+1+1$$

$$x+y+z<3$$

$$x+y+z \geq 3 \wedge (x<1 \wedge y<1 \wedge z<1)$$

$$\forall X$$

$$X \times \emptyset = \emptyset$$

sporom

Nech $X \times \emptyset \neq \emptyset$. Potom $\exists m \in X \times \emptyset \Rightarrow$
$\Rightarrow m = (x, y),\ x \in X \wedge \underline{y \in \emptyset}$ — spor

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100 lopt 9 krabíc $\Rightarrow$ v aspoň 1 krabici

bude aspoň 12 lopt

sporom

Nech v každej krabici je najviac 11 lopt. Potom spolu máme najviac 99 lopt — spor

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$$\forall n \in \mathbb{N}: 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

1^0 $n=1$ $\quad 1 \cdot 2 = \frac{1 \cdot 2 \cdot 3}{3}$

2^0 $\forall n \geq 1: V(n) \Rightarrow V(n+1)$

$$1\cdot 2+2\cdot 3+3\cdot 4+\dots+(n+1)(n+2)=$$

$$=\underbrace{1\cdot 2+2\cdot 3+3\cdot 4+\dots+n(n+1)}+(n+1)(n+2)=$$

$$=\frac{n(n+1)(n+2)}{3}+(n+1)(n+2)=(n+1)(n+2)\left[\frac{n}{3}+1\right]=$$

$$=(n+1)(n+2)\frac{n+3}{3}=\frac{(n+1)(n+2)(n+3)}{3}$$

3° $1^\circ \wedge 2^\circ \implies \forall n \geq 1: V(n)$

$$\sum_{i=1}^{n} i(i+1)=\frac{n(n+1)(n+2)}{3}$$

$$1^2+2^2+3^2+\dots+n^2=\frac{1}{6}n(n+1)(2n+1)$$

indukciou

1° 1^2 $\frac{1}{6}1(1+1)(2\cdot 1+1)=\frac{1}{6}6=1$

$\implies 1^2=\frac{1}{6}(1+1)(2\cdot 1+1)$

2° $\forall n \geq 1: V(n) \implies V(n+1)$

$$\underbrace{1^2+2^2+3^2+\dots+n^2}+(n+1)^2\overset{?}{=}\frac{1}{6}(n+1)(n+2)(2n+3)$$

$$\frac{1}{6}n(n+1)(2n+1)+(n+1)^2\overset{?}{=}\frac{1}{6}(n+1)(n+2)(2n+3)$$

$$L=\frac{1}{6}n(n+1)(2n+1)+(n+1)^2=\frac{1}{6}n\left(2n^2+n+2n+1\right)+$$

$$+n^2+2n+1=\frac{1}{6}\left[2n^3+3n^2+n\right]=\frac{1}{6}\left[6n^2+12n+6\right]=$$

$$=\frac{1}{6}\left[2n^3+9n^2+13n+6\right]$$

$$P = \frac{1}{6}(n+1)(n+2)(2n+3) = \frac{1}{6}(n+1)(2n^2+3n+4n+6) =$$
$$= \frac{1}{6}(n+1)(2n^2+7n+6) = \frac{1}{6}(2n^3+7n^2+6n+2n^2+$$
$$+7n+6) = \frac{1}{6}(2n^2+9n^2+13n+6)$$

$L = P$

$3^\circ: 1^\circ \wedge 2^\circ \Rightarrow \forall n \geq 1 : V(n)$

$\forall n \in N \qquad 5 \mid 11^n - 6$

$1^\circ \; n = 1 \qquad 11^1 - 6 = 5 \qquad 5 \mid 5$

$2^\circ \quad \forall n \geq 1 : V(n) \Rightarrow V(n+1)$

$$11^{n+1} - 6 = 11 \cdot 11^n - 6 = (10+1)11^n - 6 =$$
$$= 10 \cdot 11^n + 11^n - 6$$

$$\left.\begin{array}{l} 5 \mid 11^n - 6 \\ 5 \mid 10 \cdot 11^n \end{array}\right\} \Rightarrow 5 \mid 11^n - 6$$

$n \geq 1$

$4 \mid (6 \cdot 7^n - 2 \cdot 3^n)$

$1^\circ \; 6 \cdot 7^1 - 2 \cdot 3^1 = 42 - 6 = 36 \qquad 4 \mid 36$

$n = 1$

$$6 \cdot 7^{n+1} - 2 \cdot 3^{n+1} = 7 \cdot 6 \cdot 7^n - 3 \cdot 2 \cdot 3^n =$$
$$= (8-1)6 \cdot 7^n - (4-1) \cdot 2 \cdot 3^n = \underline{8} \cdot 6 \cdot 7^n - \underline{4} \cdot 2 \cdot 3^n -$$
$$- \underbrace{(6 \cdot 7^n - 2 \cdot 3^n)}_{4 \mid} = 4 \cdot k$$

$2n+1 \leq 2^n \qquad n \geq 3$

$1^\circ \quad n=3 \qquad 2\cdot 3+1 \stackrel{?}{\leq} 2^3 \qquad 7 \leq 8$ ✓

$2^\circ \quad \forall n \geq 3: V(n) \Rightarrow V(n+1)$

$$2^{n+1} = 2^n + 2^n \geq 2^n + 2 \geq 2n+1+2 \geq 2n+3 =$$
$$= 2(n+1)+1$$

$3^\circ: \ 1^\circ \wedge 2^\circ \Longrightarrow \forall n \geq 3: V(n)$