$\forall m,n \in \mathbb{Z}: 2|m \wedge 2|n \Rightarrow 2|(m+n)$

$2|m \wedge 2|n \Rightarrow m=2k \wedge n=2l,\ k,l\in\mathbb{Z} \Rightarrow$

$\Rightarrow m+n = 2k+2l = 2(k+l) \Rightarrow 2|(m+n)$

$\forall x,y \in \mathbb{R}: x\in\mathbb{Q} \wedge y\in\mathbb{Q} \Rightarrow x+y\in\mathbb{Q}$

$x\in\mathbb{Q} \wedge y\in\mathbb{Q} \Rightarrow x=\frac{p}{q} \wedge y=\frac{r}{s},\ p,q,r,s\in\mathbb{Z} \Rightarrow$

$\Rightarrow x+y = \frac{p}{q}+\frac{r}{s} = \frac{ps+rq}{qs} = \frac{m}{n},\ m,n\in\mathbb{Z} \Rightarrow$

$\Rightarrow x+y\in\mathbb{Q}$

X,Y,Z – množiny

$\underline{X\subseteq Y} \Rightarrow X\cup Z \subseteq Y\cup Z$

$a\in X\cup Z \Rightarrow a\in Y\cup Z$

$\underline{a\in X\cup Z} \Rightarrow a\in X \vee a\in Z$

1) $a\in X \Rightarrow a\in Y \Rightarrow a\in Y\cup Z$

2) $a\in Z \Rightarrow a\in Y\cup Z$

$\Rightarrow \underline{a\in Y\cup Z}$

$\forall n\in\mathbb{Z}: 5\nmid n^3 \Rightarrow 5\nmid n \qquad p\Rightarrow q \qquad \neg q\Rightarrow\neg p$

$\underline{5|n \Rightarrow 5|n^3}$

$\underline{5|n} \Rightarrow n=5k,\ k\in\mathbb{Z} \Rightarrow n^3=(5k)^3=5^3k^3=$

$=5\cdot(25k^3) \Rightarrow \underline{5|n^3}$

$\forall x,y,z\in\mathbb{R}: x+y+z\geq 3 \Rightarrow (x\geq 1 \vee y\geq 1 \vee z\geq 1)$

$(x<1 \wedge y<1 \wedge z<1) \Rightarrow \underline{x+y+z<3}$

$x<1 \wedge y<1 \wedge z<1 \Rightarrow x+y+z < 1+1+1 \Rightarrow$

$\Rightarrow x+y+z<3$

X

$X \times \emptyset = \emptyset$

sporom

$X \times \emptyset \neq \emptyset \Rightarrow \exists m \in X \times \emptyset \Rightarrow m = (x, y)$,

$x \in X \wedge y \in \emptyset \Rightarrow \exists y : y \in \emptyset$ — spor

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– 100 ľápt, 9 krabíc

aspoň v 1 krabici je aspoň 12 ľápt

sporom

Nech v každej krabici je najviac 11 ľápt.

Potom máme najviac $9 \cdot 11 = 99$ ľápt – spor

100 ľápt

$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ $\quad V(n)$

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

1° $n=1$ $\qquad 1 \cdot 2 = \frac{1 \cdot 2 \cdot 3}{3}$

2° $\forall n \geq 1 : V(n) \Rightarrow V(n+1)$

$\underbrace{1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1)} + (n+1)(n+2) =$

$$= \frac{n(n+1)(n+2)}{3} + (n+1)(n+2) = (n+1)(n+2)\left[\frac{n}{3}+1\right] =$$

$$= (n+1)(n+2)\frac{n+3}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

3°: $1^\circ \wedge 2^\circ \Longrightarrow \forall n \geq 1 : V(n)$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1) \qquad V(n)$$

1° $n = 1$ $\qquad 1^2 = \frac{1}{6} 1 \cdot 2 \cdot 3$

2° $\forall n \geq 1 : V(n) \Rightarrow V(n+1)$

$$\underbrace{1^2 + 2^2 + 3^2 + \dots + n^2}_{} + (n+1)^2 \stackrel{?}{=} \frac{1}{6}(n+1)(n+2)(2n+3)$$

$$\frac{1}{6} n(n+1)(2n+1) + (n+1)^2 \stackrel{?}{=} \frac{1}{6}(n+1)(n+2)(2n+3)$$

$$L = \frac{1}{6} n(2n^2 + n + 2n + 1) + n^2 + 2n + 1 =$$

$$= \frac{1}{6}(2n^3 + 3n^2 + n) + \frac{1}{6}(6n^2 + 12n + 6) =$$

$$= \frac{1}{6}(2n^3 + 9n^2 + 13n + 6)$$

$$P = \frac{1}{6}(n+1)(2n^2 + 3n + 4n + 6) = \frac{1}{6}(n+1)(2n^2 + 7n + 6)$$

$$= \frac{1}{6}(2n^3 + 7n^2 + 6n + 2n^2 + 7n + 6) =$$

$$= \frac{1}{6}(2n^3 + 9n^2 + 13n + 6)$$

$$L = P$$

3°: $1^\circ \wedge 2^\circ \Longrightarrow \forall n \geq 1 : V(n)$

$\forall n \geq 1 : 6 \mid (7^n - 1)$ $\qquad V(n)$

1° $n = 1$ $\quad 7^1 - 1 = 6 \quad 6 \mid 6$

2° $\forall n \geq 1 : V(n) \Rightarrow V(n+1)$

$$7^{n+1} - 1 = 7 \cdot 7^n - 1 = (6+1)7^n - 1 =$$

$$= \underline{6 \cdot 7^n} + \underbrace{7^n - 1}_{IP} \Longrightarrow 6 \mid (7^{n+1} - 1)$$

$/$ $\quad 4 \mid (6 \cdot 7^n - 2 \cdot 3^n) \qquad n \geq 1$

1° $n = 1$ $\quad 6 \cdot 7^1 - 2 \cdot 3^1 = 42 - 6 = 36 \quad 4 \mid 36$

2° $\forall n \geq 1 : V(n) \Rightarrow V(n+1)$

$$6 \cdot 7^{n+1} - 2 \cdot 3^{n+1} = 7 \cdot 6 \cdot 7^n - 3 \cdot 2 \cdot 3^n =$$

$$= (8-1) 6 \cdot 7^n - (4-1) 2 \cdot 3^n = 8 \cdot 6 \cdot 7^n - 4 \cdot 2 \cdot 3^n -$$

$$- 6 \cdot 7^n + 2 \cdot 3^n = \underline{8} \cdot 6 \cdot 7^n - \underline{4 \cdot 2} \cdot 3^n - (\underbrace{6 \cdot 7^n - 2 \cdot 3^n}_{IP \quad V(n)})$$

3° $1^\circ \wedge 2^\circ \Rightarrow \forall n \geq 1 : V(n)$

$2n + 1 \leq 2^n \qquad n \geq 3 \qquad V(n)$

1° $n = 3$ $\quad 2 \cdot 3 + 1 \leq 2^3 \quad 7 \leq 8$

2° $\forall n \geq 3 : V(n) \Rightarrow V(n+1)$

$$2^{n+1} = 2 \cdot 2^n = 2^n + 2^n \geq 2^n + 2 \underset{IP}{\geq} 2n + 1 + 2 =$$

$$= 2n + 3 = 2(n+1) + 1$$