WEIGHTED SHIFTS ON DIRECTED SEMI-TREES

Witold Majdak

We will introduce a concept of a directed semi-tree (which is more general that a directed tree), and generalize the notion of a weighted shift on a directed tree (recently defined and widely discussed in [3]) to the case of a directed semi-tree. We will show that such weighted shifts have similar properties to those acting on directed trees. Finally we will point out that the generalized creation operator, defined by Bargmann in [1], is unitarily isomorphic to a weighted shift acting on the so-called infinite Bargmann graph (being a directed semi-tree of a special form). The talk is based upon a joint paper with Jerzy Bartłomiej Stochel [2].

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2010 Mathematics Subject Classification 47B37.

MAPS ON CLASSES OF LINEAR OPERATORS PRESERVING MEASURE OF COMMUTATIVITY

Gergő Nagy

Several quantities can be regarded as reasonable measures of commutativity between bounded linear operators. However, probably the most natural candidate to represent this measure is the norm of the commutator of two operators. In this talk, we present some results concerning the structure of those maps on certain classes of Hilbert space operators which preserve a given unitary invariant norm of commutators. These classes are the space of self-adjoint operators, the group of unitary operators, the set of rank-one projections and the class of density operators on a given complex Hilbert space. The results of the talk are based on a joint work with György Gehér.

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The work is supported by the TÁMOP-4.2.2/B-10/1-2010-0024 project. The project is co-financed by the European Union and the European Social Fund.

ON WANDERING VECTORS FOR UNITARY OPERATORS AND ISOMETRIES

Patryk Pagacz

The talk is based upon a joint work with Z. Burdak, M. Kosiek and M. Słocinski. It concerns linear isometries V on a complex Hilbert space $\mathcal{H}$.

Definition. A vector $f \in \mathcal{H}$ is wandering vector for V iff $\langle Vnf, f^{\rangle} = 0$ for all $n \in \mathbb{N}$. At the first part of our talk we present the following theorem.

Theorem (cf. [1]). For any isometry $V \in \mathcal{B}(\mathcal{H})$ there is a unique decomposition

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_w$$

reducing operator T such, that

- $\mathcal{H}_w$ is linear span of wandering vectors,
- $\mathcal{H}_0 \subset \mathcal{H}_u$.

We describe the $\mathcal{H}_0$ part for unitary and nonunitary isometries.

In the second part we focus on wandering vectors for nonunitary isometries which projections on the unitary part and on the shift part are not trivial. We show an example of that kind of wandering vector and we presented some connected facts.

References

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ON THE κth ROOT OF A STIELTJES MOMENT SEQUENCE

Jerzy Bartłomiej Stochel

We will consider Stieltjes moment sequences $\{a_n\}_{n=0}^{\infty}$ whose κth roots $\{\sqrt[\kappa]{a_n}\}_{n=0}^{\infty}$ are Stieltjes moment sequences, where κ is a fixed integer greater then or equal to 2. Under the assumption that $\{a_n\}_{n=0}^{\infty}$ determinate, we give a formula for the (closed) support of a representing measure μ of $\{a_n\}_{n=0}^{\infty}$ written in terms of the support of a representing measure ν of $\{\sqrt[\kappa]{a_n}\}_{n=0}^{\infty}$. We provide some solutions to the following problem: given a hole of the support of the measure μ, determine the circumstances under which the support of ν has a hole and then localize it. The talk is based upon a joint paper with Jan Stochel [1].

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[1] J. Stochel, J.B. Stochel, *On the κth root of a Stieltjes moment sequence*, J. Math. Anal. Appl. **396** (2012), 786–800.

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2010 Mathematics Subject Classification 44A60.

Transformations on density operators leaving f-divergences invariant

Lajos Molnár, Gergő Nagy, Patrícia Szokol

Classical f-divergences between probability distributions were introduced by Csiszár and by Ali and Silvey, independently. These concepts, which are widely used in classical information theory and statistics, can be regarded as measure of distance or difference between probability distributions.

Recently, Hiai, Petz et al. have introduced and studied the corresponding concept of quantum f-divergence for quantum states (or density operators) in the place of probability distributions.

Let H denote a given finite dimensional Hilbert space. In this talk we present a Wigner-type result for transformations that preserve the f-divergence. Namely, for an arbitrary strictly convex function f defined on the non-negative real line we show that every transformation on the space of all density operators on H which preserves the quantum f-divergence is implemented either by a unitary or by an antiunitary operator on H.

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This research was realized in the frames of TÁMOP 4.2.4. A/2-11-1-2012-0001 "National Excellence Program - Elaborating and operating an inland student and researcher personal support system convergence program". The project was subsidized by the European Union and co-financed by the European Social Fund. The work is also supported by the TÁMOP-4.2.2/B-10/1-2010-0024 project. The project is co-financed by the European Union and the European Social Fund.

POWER CONVERGENCE OF ABEL AVERAGES

Jaroslav Zemánek

We intend to report on some parts of the recent papers [1] and [2], where we characterize the power convergence of Abel averages of operator resolvents in terms of spectrum, and also study Abel ergodicity of the resolvents.

References

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