9th Workshop Functional Analysis and its Applications, September 9–14, 2013, Nemecká, Slovakia, 33–34

# REACHABILITY FOR ZYGODACTY BIRD'S FOOT

ANNA CHIARA LAI AND PAOLA LORETI

In this talk we discuss a paper in preparation [LL2]. In this paper we establish a model for the foot of a bird with fours toes, typically observed in parrots. We are able to describe the reachability by using the theory of expansions in non-integer bases and the grasping problem.

## 1. Basic in expansions in non-integer bases

W recall basic facts in in expansions in non-integer bases

- positional number system: $(\lambda, A)$, s.t. $\mid \lambda \mid > 1$, $A \subset \mathbb{C}$; representable number $x$: there exists an expansion $(c_j)$ with $c_j \in A$ for $x$, i.e.,

$$x = \sum_{j=1}^{\infty} \frac{c_j}{\lambda^j}$$

- $\lambda = 2$, $A = \{0, 1\}$: binary expansion;
- $\lambda = 3$, $A = \{0, 2\}$: the set of representable numbers is Middle Third Cantor set.

We adapt the expansion in non-integer bases to foot rotation. We assume the foot rotation

- discrete in time: to each "clock" is associated one action;
- finite in controls: a finite numbers of controls for each phalanx.

The main idea is the following

| | | |
|---|---|---|
| base | $\leftrightarrow$ | physical properties of the digit |
| alphabet | $\leftrightarrow$ | control set |
| representability | $\leftrightarrow$ | reachability |

To describe the reachability set we will use the theory of iterated function system (IFS)

## 2. IFS

An iterated function system (IFS) is a set of contractive functions $f_j : \mathbb{C} \to \mathbb{C}$. We recall that a function in a metric space $(X, d)$ is a contraction, if for every $x, y \in X$

$$d(f(x), f(y)) < c \cdot d(x, y)$$

for some $c < 1$. Hutchinson showed that every finite IFS, namely every IFS with finitely many contractions, admits a unique non-empty compact fixed point $R$ w.r.t the Hutchinson operator

$$\mathcal{F} : \mathcal{S} \mapsto \bigcup_{j=1}^{J} f_j(\mathcal{S})$$

Moreover for every non-empty compact set $S \subseteq \mathbb{C}$

$$\lim_{k \to \infty} \mathcal{F}^k(\mathcal{S}) = \mathcal{R}.$$

The *attractor* $\mathcal{R}$ is a self-similar set and it is the only bounded set satisfying $\mathcal{F}(\mathcal{R}) = \mathcal{R}$.

---

2010 *Mathematics Subject Classification.* 15A47.

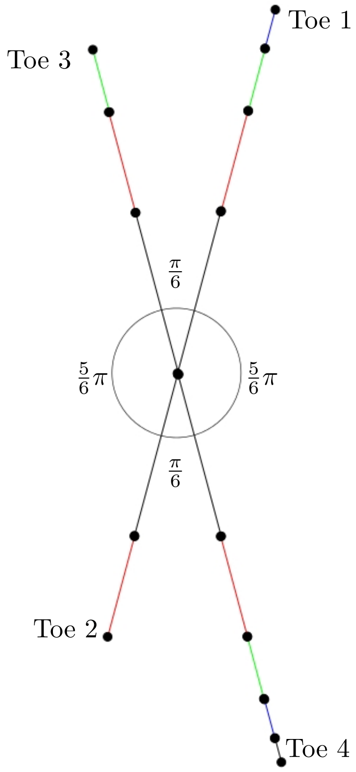


FIGURE 1. A zygodacty bird's foot. The scaling ratio $\rho$ is the Golden Mean, the angles between toes are given by setting $\omega_0 = \pi/6$.

## 3. A multi-phalanx self-similar foot

We focus on the model based on the following

**Main features of the toes**

- different number of phalanxes in the toes;
- constant ratio between phalanxes;
- Toe 1 and Toe 4 have respectively four and five phalanxes;
- Toe 2 and Toe 3 have respectively three and two phalanxes;
- phalanxes can rotate or simply do nothing;

**Main features of the foot**

- the angle between Toe 1 and Toe 2 is $\pi$;
- the angle between Toe 3 and Toe 4 is $\pi$;
- the angle between Toe 1 and Toe 3 is $\omega_0 \in (0, \pi/2)$.

A mathematical description will be given and the reachability and the grasping phenomenon analyzed. The results are based on a previous paper [LL1].

## 4. References

[LL1] Anna Chiara Lai, Paola Loreti: Robot's finger and expansions in non-integer bases. NHM 7(1): 71-111 (2012).

[LL2] Anna Chiara Lai and Paola Loreti, in preparation.

Dipartimento di Matematica, Universita' di Padova, Via Trieste, 63, 35121 Padova

Dipartimento di Scienze di Base e Applicate per l'Ingegneria, Sapienza, Universita' di Roma, via A. Scarpa n.16, 00161 Roma

*E-mail address*: `paola.loreti@sbai.uniroma1.it`