ERGODIC PROPERTIES OF OPERATORS IN SOME SEMI-HILBERTIAN SPACES

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The talk is based upon a joint work with Laurian Suciu and Nicolae-Adrian Secelean. It concerns linear operators T on a complex Hilbert space $\mathcal{H}$, which are bounded with respect to the seminorm induced by a positive operator A on $\mathcal{H}$. Our aim is to obtain some ergodic conditions for T with respect to A.

We shall assume that $A \in \mathcal{B}(\mathcal{H})$ is a positive operator (i.e., $\langle Ah, h\rangle \geq 0$ for all $h \in \mathcal{H}$). Such an A induces a positive semidefinite sesquilinear form $\langle\cdot,\cdot\rangle_A : \mathcal{H}\times\mathcal{H} \to \mathbb{C}$ defined by

$$\langle h, k\rangle_A = \langle Ah, k\rangle, \quad h, k \in \mathcal{H}.$$

Denote by $\|\cdot\|_A$ the seminorm induced by $\langle\cdot,\cdot\rangle_A$, i.e., $\|h\|_A = \sqrt{\langle h, h\rangle_A}$ for every $h \in \mathcal{H}$. We put

$$\mathcal{B}_A(\mathcal{H}) = \{T \in \mathcal{B}(\mathcal{H}) \,|\, \exists c > 0\, \forall h \in \mathcal{H}: \ \|Th\|_A \leq c\|h\|_A\}.$$

We say that an operator $T \in \mathcal{B}(\mathcal{H})$ is *A-bounded* if T belongs to $\mathcal{B}_A(\mathcal{H})$. We equip $\mathcal{B}_A(\mathcal{H})$ with the seminorm $\|\cdot\|_A$ defined as follows:

$$\|T\|_A = \sup_{h\in\overline{\mathcal{R}(A)},\, h\neq 0} \frac{\|Th\|_A}{\|h\|_A} < \infty.$$

$T \in \mathcal{B}_A(\mathcal{H})$ is *A-power bounded* if $\sup_{n\in\mathbb{N}} \|T^n\|_A < \infty$.

For $T \in \mathcal{B}_A(\mathcal{H})$, an operator $S \in \mathcal{B}_A(\mathcal{H})$ is called an *A-adjoint* of T if for every $h, k \in \mathcal{H}$

$$\langle Th, k\rangle_A = \langle h, Sk\rangle_A,$$

i.e., $AS = T^*A$.

Recall that an operator $T \in \mathcal{B}(\mathcal{H})$ is *Cesàro ergodic* if the sequence $\{M_n(T)\}_{n=1}^{\infty}$ of Cesàro averages $M_n(T) = \frac{1}{n}\sum_{j=0}^{n-1} T^j$ of T is convergent in the strong operator topology, i.e.,

$$(1) \quad \lim_{n\to\infty} \frac{1}{n}\sum_{j=0}^{n-1} T^j h = Ph, \quad h \in \mathcal{H},$$

where P is the oblique projection with $\mathcal{N}(P) = \overline{\mathcal{R}(I-T)}$ and $\mathcal{R}(P) = \mathcal{N}(I-T)$; P is called the *ergodic projection* of T. If T and T^* are Cesàro ergodic, then

$$(2) \quad \lim_{n\to\infty} \frac{1}{n}\sum_{j=0}^{n-1} T^{*j} h = P^*h, \quad h \in \mathcal{H}.$$

Both limits in (1) and (2) coincide if and only if the ergodic projection P is orthogonal (or equivalently $\mathcal{N}(I-T) = \mathcal{N}(I-T^*)$). If this is the case, then we say that T is *orthogonally mean ergodic.*

In our talk we will consider the A-adjoint and $A^{1/2}$-adjoint of T in order to obtain some new ergodic conditions for T. These operators will be employed to investigate the class of orthogonally mean ergodic operators as well as that of A-power bounded operators. We will also expand the notion of A-ergodicity to the class of A-bounded operators. Some classes of orthogonally mean ergodic or A-ergodic operators, which come from the theory of generalized Toeplitz operators will be considered.