

RECENT CONTRIBUTIONS TO OPERATOR ERGODIC THEORY

LAURIAN SUCIU

Abstract. The classical mean ergodic theorems concerning the uniform, strong or weak convergence of Cesaro and Abel averages, refer to power bounded operators in Banach spaces; for example, the results of J. von Neumann for unitary operators in Hilbert spaces and that of Lorch, Kakutani, Yoshida for power bounded operators in reflexive Banach spaces. But many ergodic facts can be extended to operators which are not necessarily power bounded, for instance, to Cesaro or Abel bounded operators, as have been shown by Hille, Krengel, Derriennic, Kornfeld, Kosek, Lin, Burlando, Zemanek etc. The present talk deals with new results regarding the uniform and strong convergence of Cesaro and Abel averages. In particular, an Abel uniform ergodic theorem is given. Also, some characterizations of the reflexivity of Banach spaces in the terms of Cesàro boundedness, are obtained. These are related to recent works of Fonf, Lin and Wojtaszczyk.

We infer also to some results related to Esterle- Katznelson- Tzafriri theorem for Cesàro means of higher order. We obtain new properties of the Cesàro means of order 0; 1 and 2, using some results of Nevanlinna and Strikwerda-Wade concerning the Kreiss resolvent condition.

Concerning the strong convergence of $M_n(T)$, P.L. Butzer and U. Westphal described the order of approximation of Px by $M_n(T)x$ for some $x \in X$, in reflexive case and T power bounded, P being the ergodic projection associated to T . In fact, their method can be applied for arbitrary Banach spaces, as they remarked. Their results regarding the discrete case can be connected to certain results of M. Lin and R. Sine concerning some averages, naturally associated to $M_n(T)$, and directly related to $M_n^{(2)}(T)$. For this reason, a purpose of this talk is to investigate the mean ergodic theorem for the Cesàro averages of higher order, from the point of view of saturation theory.

We obtain versions, or generalizations, of some results from [BW] and [LS], which lead to conclusion that all Cesàro means $M_n^{(p)}(T)$ for $p \geq 1$ are equivalent, in the sense that they have the same saturation class. We mention that a similar conclusion is also true concerning Cesaro averages in semi-hilbertian spaces, that is for A -contractions in Hilbert spaces induced by positive operators A .

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DEPARTMENT OF MATHEMATICS, "LUCIAN BLAGA" UNIVERSITY OF SIBIU, DR. ION RATIU 5-7, SIBIU, 550012, ROMANIA

E-mail address: laurians2002@yahoo.com