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PROPERTIES OF COMPOSITION OPERATORS IN L^2-SPACES VIA INDUCTIVE LIMITS

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ABSTRACT. The question of dense definiteness and boundedness of composition operators in L^2-spaces are studied by means of inductive limits of operators. We prove, by use of an inductive techniques, that assorted unbounded composition operators in L^2-spaces with matrical symbols are cosubnormal.

1. Dense deifiniteness and boundedness

One way of study unbounded operators are inductive limits of operators (cf. [6],[7]). In particular, they can be useful when dealing with unbounded composition operators in L^2-spaces. One of the aims of this talk is to discuss the questions of dense definiteness and boundedness of this class of operators. These two properties have characterization (cf. [2],[5],[9]]) which in a more concrete situations seem difficult to apply. For example, this is the case of a composition operator induced by an infinite matrix in $L^2(\mu_G)$, where μ_G is the gaussian measure on $\mathbb{R}^\infty$. One possible approach to deal with these problems is a technique based on inductive limits. Tractable criteria for the above mentioned properties will be presented. This is possible if the L^2-space (in which a given composition operator acts) is an inductive limit of L^2-spaces with underlying measure spaces forming a projective system. In this case both the dense definiteness and boundedness can be described in terms of asymptotic behaviour of appropriate Radon-Nikodym derivatives.

2. Subnormality

The inductive limit technique can also be applied when discussing the question of subnormality and cosubnormality of composition operators induced by linear transformations of $\mathbb{R}^\kappa$ (cf. [3],[8],[10]). Let γ be an entire function on $\mathbb{C}$ of the form $\gamma(z) = \sum_{n=0}^\infty a_n z^n$, for $z \in \mathbb{C}$, where a_n are nonnegative real numbers and $a_k > 0$ for some $k \geq 1$, and let $|\cdot|$ be a norm on $\mathbb{R}^\kappa$ induced by an inner product. For a given positive integer κ we define the σ-finite measure μ_γ on $\mathfrak{B}(\mathbb{R}^\kappa)$, the σ-algebra of Borel subsets of $\mathbb{R}^\kappa$, by

$$\mu_\gamma(\mathrm{d}x) = \gamma(|x|^2) m_\kappa(\mathrm{d}x),$$

where m_κ is the κ-dimensional Lebesgue measure on $\mathbb{R}^\kappa$.

It was shown in [3, Theorem 32] that a normal linear transformation A of $(\mathbb{R}^\kappa, |\cdot|)$ induces subnormal composition operator C_{A} in $L^2(\mu_\gamma)$. The proof of this fact involved a highly non-trivial construction of a measurable family of probability measures satisfying the so-called consistency condition. This fact can be proved in a different manner, based on inductive limits.

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