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SPECTRAL MAPPING THEOREMS FOR POLYNOMIALLY BOUNDED OPERATORS

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Abstract. Spectral mapping theorems are proved for residual sets and quasianalytic spectral sets of polynomially bounded operators.

We studied quasianalytic contractions in a sequel of papers [K1], [K2], [K3], [KT], [KSz1] and [KSz2]. These investigations have been extended to polynomially bounded operators in [K4]. We note that this extension is not direct consequence of the contraction case, since polynomially bounded operators are not necessarily similar to contractions; see [P]. Absolutely continuous (a.c.) polynomially bounded operators form the largest class of operators, where the H^∞-functional calculus — introduced for contractions by Sz.-Nagy and Foias (see Chapter III in [NFBK]) — can be applied (see Section 5 in [K4]). In this talk we identify the unitary asymptotes of the operators resulted by this calculus. We determine the transformation rules for local residual sets, residual sets and quasianalytic spectral sets.

Let $\mathcal{H}$ be an infinite dimensional, complex Hilbert space, and let $\mathcal{L}(\mathcal{H})$ stand for the C^*-algebra of all bounded, linear operators acting on $\mathcal{H}$. *Let $T \in \mathcal{L}(\mathcal{H})$ be an arbitrary a.c. polynomially bounded operator.* The H^∞-functional calculus $\Phi_T\colon H^\infty \to \mathcal{L}(\mathcal{H}), f \mapsto f(T)$ for T is the uniquely determined weak-$*$ continuous (unital) representation of the Banach algebra H^∞, which transforms the identical function $\chi(z) = z$ into T. The norm of Φ_T coincides with the polynomial bound

$$K_T := \sup\{\|p(T)\|/\|p\|_\infty : 0 \not\equiv p \in \mathcal{P}(\mathbb{T})\},$$

where $\mathcal{P}(\mathbb{T})$ denotes the restrictions of polynomials to the unit circle $\mathbb{T} = \{\zeta \in \mathbb{C} : |\zeta| = 1\}$. We recall that the Hardy class H^∞ consists of the bounded analytic functions defined on the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. For any $f \in H^\infty$, the radial limit $\check{f}(\zeta) := \lim_{r\to 1-} f(r\zeta)$ exists at almost every $\zeta \in \mathbb{T}$, with respect to the normalized Lebesgue measure m on $\mathbb{T}$. Via the identification $f \equiv \check{f}$, H^∞ can be considered as a weak-$*$ closed subspace of the dual $L^\infty(\mathbb{T}) := L^\infty(m)$ of the Banach space $L^1(\mathbb{T})$.

Let us consider the unilateral shift S defined on the Hardy–Hilbert space H^2 by $Sf = \chi f$. Since Φ_S is an isometry, the operator $g(S)$ is power bounded exactly when $\|g\|_\infty \leq 1$. Furthermore, $g(T) = cI$ if $g(z) = c$ for all $z \in \mathbb{D}$. *Let us assume that $g \in H^\infty$ is a non-constant function and $\|g\|_\infty \leq 1$.* Our aim is to detect the properties of the operator $Q := g(T) \in \mathcal{L}(\mathcal{H})$.

Given any $f \in H^\infty$, the composition $f \circ g \in H^\infty$ is defined by $(f \circ g)(z) := f(g(z))$ for all $z \in \mathbb{D}$.

Proposition 1. *Q is an a.c. polynomially bounded operator, and*

$$f(Q) = (f \circ g)(T)$$

holds for every $f \in H^\infty$.

Furthermore, the following statement is proved among others.

Theorem 2. *If T is quasianalytic, then so is Q, and the quasianalytic spectral set $\pi(Q)$ of Q is the measurable essential range of the restriction of g to the set $\pi(T) \cap \Omega(g)$.*

Here $\Omega(g) := \{\zeta \in \mathbb{T} : |g(\zeta)| = 1\}$.

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