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ON k–REFLEXIVITY, k–HYPERREFLEXIVITY AND HYPOREFLEXIVITY OF POWER PARTIAL ISOMETRIES

KAMILA PIWOWARCZYK AND MAREK PTAK

Let $\mathbf{B}(\mathcal{H})$ denote the algebra of bounded linear operators on a complex separable Hilbert space $\mathcal{H}$. For an operator $T \in \mathbf{B}(\mathcal{H})$ by $\mathcal{W}(T)$ we denote the smallest unital WOT-closed subalgebra of $\mathbf{B}(\mathcal{H})$ containing the operator T. If $\mathcal{W} \subset \mathbf{B}(\mathcal{H})$ is an algebra with identity by $\operatorname{Lat}\mathcal{W}$ we denote the set of all orthogonal projections onto closed subspaces of $\mathbf{B}(\mathcal{H})$ which are invariant for all operators in $\mathcal{W}$, i.e.

$$\operatorname{Lat}\mathcal{W} = \{\mathcal{L} \subset \mathcal{H} : A\mathcal{L} \subset \mathcal{L} \quad \text{for all} \quad A \in \mathcal{W}\}.$$

By Alg Lat $\mathcal{W}$ we denote the algebra of operators which leave invariant all subspaces from Lat $\mathcal{W}$, i.e.

$$\text{Alg Lat}\,\mathcal{W} = \{A \in \mathbf{B}(\mathcal{H}) : \operatorname{Lat}\mathcal{W} \subset \operatorname{Lat} A\}.$$

The algebra $\mathcal{W}$ is *reflexive* ([9]) if

$$\mathcal{W} = \text{Alg Lat}\,\mathcal{W}.$$

A single operator $A \in \mathbf{B}(\mathcal{H})$ is reflexive if $\mathcal{W}(A)$ is reflexive.

For a given operator $A \in \mathbf{B}(\mathcal{H})$ we may consider the usual distance from A to an algebra $\mathcal{W} \subset \mathbf{B}(\mathcal{H})$ (denoted by $\operatorname{dist}(A, \mathcal{W})$), but also we can define the distance determined by invariant subspaces, i.e.

$$\alpha(A, \mathcal{W}) = \sup\{\|(I-P)AP\| : P \in \operatorname{Lat}\mathcal{W}\}.$$

Usually $\alpha(A, \mathcal{W}) \le \operatorname{dist}(A, \mathcal{W})$. The operator $T \in \mathbf{B}(\mathcal{H})$ is called *hyperreflexive* if the usual distance can be controlled by the distance α ([1]), i.e. there is a positive constant κ such that

$$\operatorname{dist}(A, \mathcal{W}(T)) \le \kappa\ \alpha(A, \mathcal{W}(T)) \text{ for all } \ A \in \mathbf{B}(\mathcal{H}).$$

Longstaff characterized a reflexive subspace using rank one operators in its preannihilator ([8]). Arveson proved that the distance α may be calculated using rank one operators ([3]).

The concepts of k–reflexivity and k–hyperreflexivity are natural genaralizations of reflexivity and hyperreflexivity, namely in corresponding conditions rank one operators are replaced by operators of rank at most k ([2], [7], [6]).

We may also consider weaker property than reflexivity, namely hyporeflexivity ([4]). We say that a commutative algebra $\mathcal{W} \subset \mathbf{B}(\mathcal{H})$ is *hyporeflexive* if

$$\mathcal{W} = \mathcal{W}' \cap \text{Alg Lat}\,\mathcal{W},$$

where $\mathcal{W}'$ denotes the commutant of $\mathcal{W}$.

Recall that an operator $V \in \mathbf{B}(\mathcal{H})$ is called a *partial isometry* if V^*V is an orthogonal projection. An operator V is a *power partial isometry* if V^n is a partial isometry for every positive integer n. It is known ([5]) that if V is a power partial isometry on $\mathcal{H}$ then there is a unique orthogonal decomposition $\mathcal{H} = \mathcal{H}_u(V) \oplus \mathcal{H}_s(V) \oplus \mathcal{H}_c(V) \oplus \mathcal{H}_t(V)$ where $\mathcal{H}_u(V)$, $\mathcal{H}_s(V)$, $\mathcal{H}_c(V)$, $\mathcal{H}_t(V)$ reduce V and $V_u = V|_{\mathcal{H}_u(V)}$ is a unitary operator, $V_s = V|_{\mathcal{H}_s(V)}$ is a unilateral shift of arbitrary multiplicity, $V_c = V|_{\mathcal{H}_c(V)}$ is a backward shift of arbitrary multiplicity and $V_t = V|_{\mathcal{H}_t(V)}$ is (possibly infinite) direct sum of truncated shifts.

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The main result of my talk will be

Theorem 1. *Let $V \in \mathbf{B}(\mathcal{H})$ be a power partial isometry. Then*
(1) *V is 2–reflexive,*
(2) *V is 2–hyperreflexive,*
(3) *V is hyporeflexive.*

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Department of Applied Mathematics, University of Agriculture in Krakow, ul. Balicka 253c, 30–198 Kraków, Poland
E-mail address: kamila.piwowarczyk@ur.krakow.pl