ISOMETRIES AND ISOMORPHISMS ON POSITIVE DEFINITE MATRICES

Lajos Molnár

In this talk we give a survey on recent results that have been obtained in the past few years concerning the structures of isometries and generalized isometries (maps which preserve generalized distance measures or divergences) on positive definite matrices. We show that certain large classes of such transformations are closely related to some sorts of (non-linear) algebraic isomorphisms (Jordan triple isomorphisms or order isomorphisms) of the positive definite cones in matrix algebras. The metrics we consider cover important Riemannian and Finslerian metrics and the generalized distance measures include important Bregman and Jensen divergences.

Bolyai Institute, University of Szeged, and MTA-DE "Lendület" Functional Analysis Research Group, Institute of Mathematics, University of Debrecen, Hungary, molnarl@math.u-szeged.hu

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MEAN ERGODIC THEOREM FOR POLYNOMIAL SUBSEQUENCES

Vladimír Müller

Let T be a power bounded Hilbert space operator and let p be a polynomial satisfying $p(\mathbf{N}) \subset \mathbf{N}$. Then the Cesaro sums $N^{-1}\sum_{n=1}^{N} T^{p(n)}$ converge in the strong operator topology. This generalizes known results for unitary operators and Hilbert space contractions.

The method can be used also for other polynomial-type sequences and for bounded strongly continuous semigroups of operators.

(joint work with A.F.M. ter Elst)

Mathematical Institute, Academy of Sciences of the Czech Republic,Žitná 25, 115 67 Praha 1, Czech Republic, muller@math.cas.cz

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QUASINORMAL AND COMPOSITION OPERATORS IN L^2-SPACES

Paweł Pietrzycki

It is proved that a closed densely defined operator A is quasinormali if and only if the equality $A^{*n}A^n = (A^*A)^n$ holds for $n = 2, 3$. We will construct examples of bounded, non-quasinormal operator A which satisfies equality $A^{*n}A^n = (A^*A)^n$. An example of such an operator is given in the class of weighted shifts on directed trees. What is important, the directed tree used in the construction is rootless and therefore the operator in example is unitarily equivalent to a composition operator in L^2-space.

Main theorem *Let n be an integer greater than or equal to 2. Then there exists an injective non-quasinormal weighted shift $A \in \boldsymbol{B}(\ell^2(V_\infty))$ on the directed tree $\mathcal{T}$ which satisfies the condition $A^{*n}A^n = (A^*A)^n$. Moreover, there exists an injective non-quasinormal composition operator C in L^2-space over a σ-finite measure space satisfying the condition $C^{*n}C^n = (C^*C)^n$.*

[1] P. Pietrzycki, *The single equality $A^{*n}A^n = (A^*A)^n$ does not imply the quasinormality of weighted shifts on rootless directed trees*, http://arxiv.org/abs/1502.06396

Jagiellonian University, Kraków, Poland, pawel.pietrzycki@im.uj.edu.pl

CONTRACTIVE POLYNOMIALS IN THE VOLTERRA OPERATOR

A.F.M. ter Elst* and Jaroslav Zemánek**

If f is a holomorphic function with $f(0) = 1$ then we present sufficient conditions and necessary conditions such that $\|f(V)\| = 1$, where V is the classical Volterra operator. In particular we show that this may happen for certain polynomials f, disproving a conjecture by Lyubich and Tsedenbayar.

*Department of Mathematics, University of Auckland, Private bag 92019, Auckland 1142, New Zealand, terelst@math.auckland.ac.nz
**Institute of Mathemaics, Polish Academy of Sciences, Śniadeckich 8, 00-656 Warsaw, Poland
zemanek@impan.pl